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VALFELLS, Agust, 1934–
PRELIMINARY DESIGN OF A POWER GENERATING SYSTEM USING A FISSILE GAS.

Iowa State University of Science and Technology
Ph.D., 1962
Physics, nuclear

University Microfilms, Inc., Ann Arbor, Michigan

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1962

PRELIMINARY DESIGN OF A POWER GENERATING SYSTEM USING A FISSILE GAS

by

Agust Valfells

A Dissertation Submitted to the
Graduate Faculty in Partial Fulfillment of
The Requirements for the Degree of
DOCTOR OF PHILOSOPHY

Major Subject: Nuclear Engineering

Approved:

Signature was redacted for privacy.

In Charge of Major Work

Signature was redacted for privacy.

Head of Major Department

Signature was redacted for privacy.

Dean of Graduate College

Iowa State University
Of Science and Technology

Ames, Iowa

1962

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I. INTRODUCTION

A. Purpose of the Study

This thesis is intended to be a feasibility study of a system that utilizes a fissile gas, thermally ionized by its own fission, for electric power generation by magnetohydrodynamic means.

It is an effective continuation of some work undertaken previously by the author, originally concerned with investigating the effects of a magnetic field on a hydrogen-oxygen flame, which subsequently led to a theoretical investigation of the possibility of a device such as described above. The theoretical study indicated the possibility of such a device and led to the preliminary design of the specific system which is the topic of this thesis.

B. Results of Literature Survey

The concept of thermal ionization grew out of the study of hot gases. Until about 1900, when electrical techniques for energizing gases were developed, the only laboratory source of a hot gas was a flame. Therefore, most of the early work concerned with ionization in a hot gas was limited to flame studies. Gaydon (1) refers to Volta as having been one of the first to observe the electrical conductivity of flames, when, in 1801, he noticed that a flame will discharge a charged nonconductor. Shortly afterwards, in 1802, Erman (2) succeeded in obtaining minute currents from flames. In 1827 Pouillet (3) noticed that an electric field will tend to separate the inner and outer cones of a flame, as indicated in Fig. 1. Various other workers, such as Arrhenius, investigated the conductivity and other electrical properties of flames in the latter half of the nineteenth century. The first to point out that flame conductivity might be the result of ionization was probably Giese (4), in 1877.

Probably the first extensive work on electrical flame phenomena was that undertaken by Wilson (5), who published a book on his studies, wherein a chapter is devoted to the Hall effect in flames. Marx (6) had also made a study of the Hall effect in flames as early as 1900. Thomson and Thomson (7) had formulated a theory of this same effect, based on the microscopic point of view.

Wilson's results made it possible to calculate the mobilities of the charges present in a flame. These were found to be of the order of 1 cm/sec per unit potential gradient for the positive charges and about a thousand times greater for the negative ones.

This suggests that the negative charges are free electrons and that the positive ones have sizes corresponding to the ions of the lighter elements.

The charge separation taking place in a flame can thus be explained, as the positive ions with their limited mobilities tend to be swept away from the flame front into the outer cone of the flame along with the combustion products, whereas the electrons with their small collision diameter are more or less free to diffuse throughout the entire flame. This results in a net positive charge for the outer cone relative to the inner one.

Becker (8) has published an extensive review of most of the work done on electrical flame phenomena up to about 1929. He also found that the potential gradient of a voltage impressed across a flame varies as indicated in Fig. 2. It should be noted that the potential gradient is greatest near the electrodes, i.e., the flame has the greatest electrical resistance in these regions. As the flame is coldest in the vicinity of the electrodes this suggests that thermal ionization takes place in the flame. This view was suggested by Saunders (9) in 1926.

In 1920 Saha (10) published his theory on thermal ionization in connection with studies of star spectra. This rendered possible a more quantitative treatment of ionization phenomena.

Although mechanisms other than thermal ionization can cause ionization of flames at low temperatures ($\approx 10^3$ K), as has been shown by Marsden (11), Saha's theory predicts that at higher temperatures ($> 3 \times 10^3$ K) thermal ionization must become the prevalent mechanism of ion formation.

A recapitulation of the foregoing will show that it was noticed early that flames possess electrical properties, such as conductivity and being affected by an electric field. Considerable work on flame conductivity was undertaken in the late nineteenth and early part of this century.

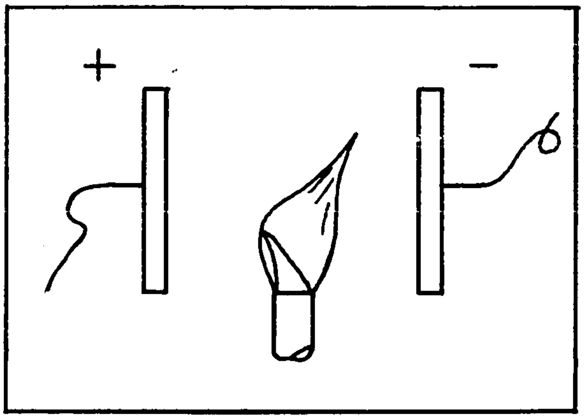


Fig. 1. A flame in an electric field

Explanations for the conductivity were first offered by Giese in 1877 and after the publication of Saha's theory in 1920 the electrical properties of flames were understood to be, to a large extent, due to thermal ionization.

After 1900 electrical techniques for energizing gases became available, and much higher temperatures were obtainable than with flames.

Relatively little investigation concerning thermally ionized gases was carried out in the period between 1930 and 1950.

After 1950, due to greatly renewed interest in combustion, for jet propulsion purposes, there was again increased interest in thermal ionization. Much of this work was carried out under the auspices of project SQUID (12).

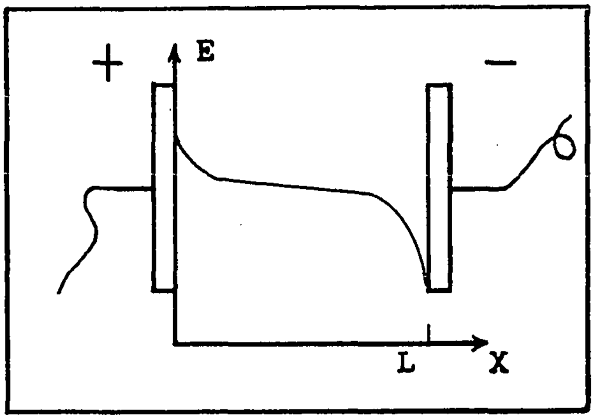


Fig. 2. The potential gradient in a flame

During this period, however, the important field of magnetohydrodynamics was being developed by such pioneering workers as Alfven (13), Cowling (14), Spitzer (15) and Chandrasekhar (16) in connection with astrophysical investigations.

After 1954, in connection with project Sherwood, the attempt to harness fusion energy, the activity in this field has been greatly intensified. Bishop's (17) book on Project Sherwood is a good source of further references in this regard.

Finally the development of magnetohydrodynamics has made possible its application to the analysis of electric power generating devices (magnetohydrodynamic generators) wherein the classical solid conductor moving through a magnetic field is replaced by a high velocity stream of a moving ionized gas.

A few such devices have been suggested in recent years and for a short survey of progress in this field the reader is referred to an article by Lindley (18).

As contrasted with the macroscopic approach of magnetohydrodynamics, gas conductivity studies of partly ionized gases must also be studied from a microscopic point of view. Brown (19) gives a comprehensive summary of work done on this aspect of the problem. The theoretical treatment of the interaction of low-energy electrons with atoms was pioneered by Allis and Morse (20). Massey (21) discusses the application of their theory to electron-molecule interactions.

With the developments mentioned above taking place concurrently with the development of nuclear fission as a source of power, nuclear reactors seem promising as heat sources for magnetohydrodynamic generators when contrasted with chemically powered heat sources.

The idea then naturally suggests itself to use as the working fluid in the generator a gas which itself is fissile, in order to simplify the heat transfer and materials problems at the high temperatures required for the operation of such a device.

Gaseous core reactors have received considerable consideration in connection with the nuclear rocket program. Meghreblian (22), Rom (23) and Gray (24) all describe some of the concepts that have been forwarded for gaseous core reactors.

As mentioned before some limited studies have previously been made by the author (25, 26) of a system consisting of a magnetohydrodynamic generator powered by a gaseous phase fission reactor.

II. THEORY

A. The Concept of Thermal Ionization

According to Saha's theory the thermal ionization of a gas may be treated like a conventional reversible chemical reaction:

$$A \rightleftarrows A^{+} + e^{-}$$

The methods of thermodynamics can then be applied as to any other chemical equilibrium to calculate the equilibrium constant, K_p. Let x be the fraction of A ionized, then:

$$K_p = \frac{x^2}{1-x^2}\,p \qquad 1$$

where p is the pressure in the system in atm.

Saha derives the equation for K_p:

$$\log K_p = -\frac{U}{4.573T} + \frac{5}{2}\log T - 6.49 + \log\frac{g_e g_i}{g_a} \qquad 2$$

where,

U is the heat of ionization in cal/mole,

T is the temperature in degrees Kelvin,

K_p is the equilibrium constant in atm.

The last term is a quantum mechanical correction factor. g refers to the total number of energetically equivalent ground states of the particle in question, with the subscripts e, i and a referring to the electron, ion and neutral atom, respectively. U, the heat of ionization in cal/mole is related to the ionization potential, I, in volts, by the expression:

$$U = 23050 \, I \qquad 3$$

Solving for x from equation 1:

$$x = \sqrt{\frac{K_p}{p + K_p}} \qquad 4$$

The kinetics for recombination will be those of a second order reaction. Letting n_+ and n_- signify the particle concentration of positive and negative particles, respectively, then:

$$\frac{dn_+}{dt} = \frac{dn_-}{dt} = -\alpha n_- n_+ = -\alpha n_-^2 \qquad 5$$

as $n_- = n_+$. α is the rate constant. When the equation is solved for n_-, the result is:

$$n_- = \frac{[n_-]_{t=0}}{1 + \alpha t \, [n_-]_{t=0}} \qquad 6$$

Once a gas has been thermally ionized it will be affected by the presence of electric and magnetic fields. In order to analyze its behavior it must be studied from the microscopic as well as the macroscopic point of view.

B. The Conductivity of a Partly Ionized Gas

Consider a stream of a moving, partly ionized gas, where the ions are uniformly distributed throughout the stream.

The force exerted by a magnetic field, $\vec{B}$, on a particle of charge e that has a velocity v with respect to the field, is given by the relation:

$$\vec{F} = \vec{v} x e \vec{B} \quad \text{(in a right-handed coordinate system)} \qquad 7$$

In the absence of other effects the particle will simply gyrate around a magnetic line of force with an orbital velocity v. The angular frequency of gyration (the so-called cyclotron or gyromagnetic frequency) can be found by equating the centrifugal acceleration v^2/a, where a is the orbit radius, with the acceleration due to the magnetic field:

$$\frac{v^2}{a} = \frac{Bev}{m} \qquad 8$$

where m is the particle mass; hence,

$$f_c = \frac{\omega_c}{2\pi} = \frac{v}{2\pi a} = \frac{Be}{2\pi m} \qquad 9$$

where,

f_c is the cyclotron frequency

ω_c is the corresponding angular velocity

In a partly ionized gas there will, however, be neutral particles which will collide with the charged ones. Consider an infinite slab geometry where the gas velocity vector is perpendicular to the magnetic field vector, such as is illustrated in Fig. 3. If the gas stream velocity is V then this will be the average velocity of the neutral particles and, consequently, also that of the charged ones, (except at very low pressures).

Upon collision a charged particle will shift its center of gyration to another line of force. A succession of such collisions will cause a gradual drift in the direction perpendicular to both $\vec{V}$ and $\vec{B}$. Oppositely charged particles will drift in opposite directions (See Fig. 3).

This will give rise to a current within the gas that can be used to cause a potential gradient due to accumulation of charge at the gas boundaries. The electric field thus established will act on a charged particle as described by the equation:

$$\vec{F} = e\nabla E \qquad 10$$

Thus the total force on the particle will be:

$$F = e(\nabla E + \vec{v}x\vec{B}) \qquad 11$$

When an aggregate of particles is considered, the effect of the random velocities due to thermal agitation will cancel out and only the net effect due to the average velocity V will manifest itself. The electric field will build up until the electric and magnetic forces on the particles are balanced. The criterion for such an equilibrium is:

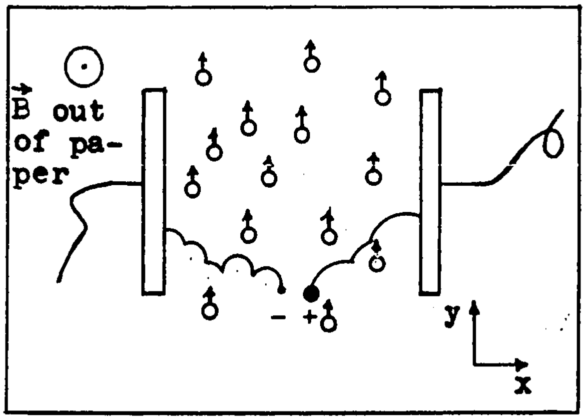


Fig. 3. Transport of ions in a partly ionized gas moving in a magnetic field

$$e \nabla E = -\vec{V}xe\vec{B} \qquad 12$$

When equilibrium is attained the particles will continue undeflected through the magnetic field. The potential gradient may be considered essentially uniform (corresponding to no space charge, as may be seen from Poisson's equation). Thus ∇E may be replaced by E/L, where L is the slab thickness, and hence,

$$E = BLV \tag{13}$$

The above equation is the familiar induction formula and represents the limiting potential that can be established across the gas.

When this situation prevails, there is no current flowing through the gas, and, conversely, the current will be at a maximum when $E = 0$, i.e., there is no potential to countereffect the magnetic field.

Next, the effect of the viscous drag forces on the motion of the charged particles within the gas must be considered. These are the result of collisions with the gas molecules, and encounters of the charged particles with one another. The macroscopic effect of the drag forces manifests itself as electrical resistance.

Consider first a free electron in a slightly ionized gas. Let v be the magnitude of electron velocity, n the gas molecular density and σ the corresponding cross section for an electron-molecule collision. As the electron mass is very small in comparison with that of the ions and gas molecules, the magnitude of the electron velocities will be very much larger than that of the other particle velocities. Hence, as a first approximation, it may be assumed that the electron velocity, v, can also be regarded as its velocity relative to the ions and the gas.

Furthermore, in a gas that is only slightly ionized (less than 1 part per 10^5), encounters between electrons and gas molecules will greatly predominate over encounters between electrons and ions and the effect due to the latter may be left out without appreciable error. Then the collision frequency, ν_c, can be expressed as:

$$\nu_c = n\sigma v \qquad 14$$

In a unidirectional force field, such as an electric field or an equivalent magnetic field where the collision frequency is very great in comparison with the cyclotron frequency, Newton's second law may be applied to the electron moving in the direction of the field:

$$\vec{F} = d(mv)/dt + \nu_c \overline{mv(1-\cos\theta)} \qquad 15$$

θ represents the scattering angle of the electron and m its mass. The first term on the right-hand side of the equation represents the inertial force due to the acceleration of the particle by the field, whereas the latter term is the average value of the viscous drag force in the field direction. It is convenient to introduce a symbol for the frequency of collision for momentum transfer as defined by the equation:

$$\nu_m = \nu_c \overline{(1-\cos\theta)} \qquad 16$$

If the assumption is made that v is constant, (i.e., that the electron is not accelerated in the field direction but that all the work done on it is expended in overcoming the viscous drag forces), then the equation may be solved for v, yielding the expression:

$$v = \frac{F}{m\,\nu_m} \qquad 17$$

Now the electric current density due to the electrons, j_-, will be:

$$j_- = n_- e v \qquad 18$$

Hence, replacing F in equation 17 by $e\nabla E$ and making the approximate substitution for v,

$$j_- = \frac{n_- e^2 \nabla E}{m\,\nu_m} \qquad 19$$

For low energy electrons the average scattering angle in electron molecule collisions is generally close to 90°. Therefore, $\nu_m = \nu_c$, which in turn may be replaced by $n\sigma v$. The conductivity, η, is equivalent to $j/\nabla E$; therefore:

$$\eta = \frac{e^2}{mv}\,\frac{n_-}{n\sigma} \qquad 20$$

This last equation represents the conductivity of a partly ionized gas that is due to the free electrons. A corresponding treatment for the positive ions will yield the ion conductivity. However, because the mobility of these is very much more restricted than that of the electrons, the total conductivity is found to be almost exclusively due to the latter (except for very rarefied gases).

Furthermore, the above equation is not rigorous, as the electron-ion interactions were ignored, but this is permissible as long as the degree of ionization is less than about 10^{-5}.

C. Magnetic Effects on the Conductivity

Consider a partly ionized gas as a mixture of three different gases, namely, the neutral gas, the electron gas and the ion gas. For the sake of simplicity, only the case where the atoms and the ions are of a similar kind will be considered. In the case of more than one kind of gas being present in the system, the treatment is essentially the same, whereas the magnitude of the drag forces will be somewhat different.

Let the mass as a whole have a velocity $\vec{v}$, the ions a velocity $\vec{v} + \vec{V}_+$, where $\vec{V}_+$ is their average velocity relative to the mass as a whole, and, similarly, let the electrons have an average velocity $\vec{V}$ with respect to the ions and therefore the total velocity of $\vec{v} + \vec{V}_+ + \vec{V}$. If the mass of the electrons is neglected, the momentum per unit volume of the mass as a whole is seen to be $(n_+ + n)m_+\vec{v}$; therefore, the neutral gas velocity is $\vec{v} - n_+\vec{V}_+/n$. It is convenient to replace the magnetic flux density, $\vec{B}$, by $\mu\vec{H}$, where μ is in this case the permeability of the gas and $\vec{H}$ the magnetic field intensity. Let ν represent the collision frequency of electrons and ions and ν_- and ν_+ represent the collision frequency of each of these with neutral gas atoms. Let $\vec{j}$ refer to current density, as before, and ρ to mass density. Consider first the electron gas. The forces accelerating it are:

a. the partial pressure gradient $-\nabla p_e$,

b. gravity, $n_-m_-\vec{g}$, where $\vec{g}$ is the gravitational vector,

c. electric and magnetic forces: $-n_-e\{\nabla E' + \mu(\vec{V} + \vec{V}_+)\times\vec{H}\}$, where $\nabla E' = \nabla E + \mu\vec{v}\times\vec{H}$,

d. drag forces. On collision with ions or neutral atoms an electron loses all its velocity with respect to the corresponding gas, on the average.

Because the mass of the electrons is so small the inertial and gravitational forces may be ignored. The drag forces due to collisions with ions will be $-n_-m_-\vec{V}\nu$ and those due to the neutral gas $-n_-m_-\{\vec{V} + \vec{V}_+(1 + n_-/n)\}\nu_-$. Thus the equation of motion of the electron gas will be:

$$\begin{aligned} 0 = -\nabla p_- - n_-e\{\nabla E' + \mu(\vec{V} + \vec{V}_+)x\vec{H}\} - n_-m_-\vec{V}\nu \\ -n_-m_-\{\vec{V} + \vec{V}_+(1 + n_-/n)\}\nu_- \end{aligned} \qquad 21$$

Let $\vec{j} = \vec{j}_-$ equal the current density due to the electrons:

$$\vec{j} = -n_-e\vec{V} \qquad 22$$

similarly for the ions:

$$\vec{j}_+ = n_+e\vec{V}_+ \qquad 23$$

then,

$$\begin{aligned} 0 = -\nabla p_- - n_-e\,\nabla E' + \mu(\vec{j} - \vec{j}_+)x\vec{H} + \mu(\omega/\nu)^{-1}H\vec{j} \\ + \mu(\omega/\nu_-)^{-1}H\{\vec{j} - \vec{j}_+(1 + n_-/n)\} \end{aligned} \qquad 24$$

where $\omega = \mu eH/m_-$ and denotes the angular velocity corresponding to the cyclotron frequency of the electrons. For brevity, let $(\omega/\nu)^{-1}$ be replaced by κ, and f denote the fraction of the gas not ionized (= 1-x).

Then,

$$1 + n_-/n = f^{-1} \qquad 25$$

hence,

$$0 = -\nabla p_- - n_- e \nabla E' + \mu(\vec{j} - \vec{j}_+) \times \vec{H}$$
$$+ \mu(\kappa + \kappa_-) H \vec{j} - \mu \kappa_- f^{-1} H \vec{j}_+ \qquad 26$$

The equation of motion for the ion gas is obtained in a similar manner, except in this case, inertial and gravitational forces must be taken into account. Drag forces will also be different. The drag force due to encounters with electrons will be equal and opposite to the corresponding force exerted on the latter. On the average, the ions will only lose half their velocity in collisions with the neutral gas atoms (or molecules if they are molecular ions). The mass acceleration is taken to be $\rho_+(d\vec{v}/dt)$, $\vec{V}_+$ being supposed small in comparison with $\vec{v}$.

Accordingly, the equation of motion for the ion gas will be:

$$\rho_+(d\vec{v}/dt) = -\nabla p_+ + \rho_+ \vec{g} + n_+ e(\nabla E' + \mu \vec{V}_+ \times \vec{H})$$
$$+ n_- m_- \vec{V} \nu - \tfrac{1}{2} n_+ m_+ \vec{V}_+ (1 + n_+/n) \nu_+ \qquad 27$$

In this case let $K_+ = (2\omega_+/\nu_+)^{-1}$ and as $\rho_+ = (1-f)\rho$, then:

$$(1-f)\rho(d\vec{v}/dt) = -\nabla p_+ + (1-f)\rho\vec{g} + n_- e\nabla E' + \mu\vec{j}_+ \times \vec{H} - \mu K H\vec{j} - \mu K_+ f^{-1} H\vec{j}_+ \qquad 28$$

The equation of motion for the gas as a whole is:

$$\rho(d\vec{v}/dt) = -\nabla p + \rho\vec{g} + \mu\vec{j}\times\vec{H} \qquad 29$$

$\vec{j}_+$ may be eliminated between equations 26 and 28. By adding them:

$$(1-f)\rho(d\vec{v}/dt) = -\nabla(p_- + p_+) + (1-f)\rho\vec{g} + \mu\vec{j}\times\vec{H} + \mu K_- H\vec{j} - \mu(K_- + K_+) f^{-1} H\vec{j}_+ \qquad 30$$

Then, by use of equation 29, $d\vec{v}/dt$ may be eliminated; then, since $p_- = p_+ = (1-f)p/(2-f)$, the result will be:

$$(K_- + K_+) f^{-1} H\vec{j}_+ = -f\mu^{-1}\nabla p_- + f\vec{j}\times\vec{H} + K_- H\vec{j} \qquad 31$$

Subsequent substitution for $\vec{j}_+$ from equation 26 will yield the expression:

$$n_- e\nabla E' + (1-f\beta)\nabla p_- = \mu(K + \beta K_+)H\vec{j} + \mu(1-2f/\beta)\vec{j}\times\vec{H} + f^2 H^{-1}(K_- + K_+)^{-1}\left\{\nabla p_- \times \vec{H} - \mu(\vec{j}\times\vec{H})\times\vec{H}\right\} \qquad 32$$

where,

$$\beta = \kappa_-/(\kappa_- + \kappa_+) \qquad 33$$

Equation 32 describes the electric behavior of a partly ionized gas in the presence of magnetic and electric fields. It is interesting to note, however, that even in the absence of these, there may still exist a current within the gas as a result of the pressure gradient. This is because of the greater mobility of the electrons as compared to the ions. Consequently, they tend to outrun the ions under a pressure gradient and thus, a net current will result. This, in turn, causes a space charge to accumulate at the gas boundaries that will effect a counterpotential that, in turn, prevents the leakage of electrons from the gas, thus maintaining its overall neutrality.

When there is no partial pressure gradient in the region of gas under consideration equation 32 reduces to:

$$\begin{aligned} n_- e \nabla E' &= \mu(\kappa + \beta\kappa_+)H\vec{j} + \mu(1-2f\beta)\vec{j}x\vec{H} \\ &\quad + f^2 H^{-1}(\kappa_- + \kappa_+)^{-1}\left\{ -\mu(\vec{j}x\vec{H})x\vec{H} \right\} \qquad 34 \end{aligned}$$

Now let:

$$\begin{aligned} A &= \mu(\kappa + \beta\kappa_+) \\ C &= \mu(1-2f\beta) \qquad 35 \\ D &= \mu f^2(\kappa_- + \kappa_+)^{-1} \end{aligned}$$

then,

$$n_- e \nabla E' = AH\vec{j} + C\vec{j}\times\vec{H} - DH^{-1}(\vec{j}\times\vec{H})\times\vec{H} \qquad 36$$

In the infinite slab geometry referred to before, the induced current will be in a plane perpendicular to the magnetic field vector.

Upon examination of Fig. 4 it is easily seen that with the infinite slab geometry $\vec{j}\times\vec{H}$ is equivalent to $-iH\vec{j}$ and $(\vec{j}\times\vec{H})\times\vec{H}$ may be replaced by $-iH(-iH\vec{j}) = i^2H^2\vec{j} = -H^2\vec{j}$. Making these replacements in equation 36 yields:

$$n_- e \nabla E' = \left\{ (A+D) - iC \right\} H\vec{j} \qquad 37$$

Therefore,

$$\vec{j} = \frac{n_- e \nabla E'}{H} \; \frac{(A+D)+iC}{(A+D)^2+C^2} \qquad 38$$

Now, since $\omega_- \gg \omega_+$, assume $\omega_+ = 0$; then,

$$\lim_{\omega_+ \to 0} \kappa_+ = \lim \nu_+/2\omega_+ = \infty \qquad 39$$

$$\lim_{\kappa_+ \to \infty} \beta = \lim \kappa_-/(\kappa_- + \kappa_+) = 0 \qquad 40$$

$$\lim_{K_+ \to \infty} \beta K_+ = \lim \frac{K_- K_+}{K_- + K_+} = \lim \frac{K_-}{1} = K_- \qquad 41$$

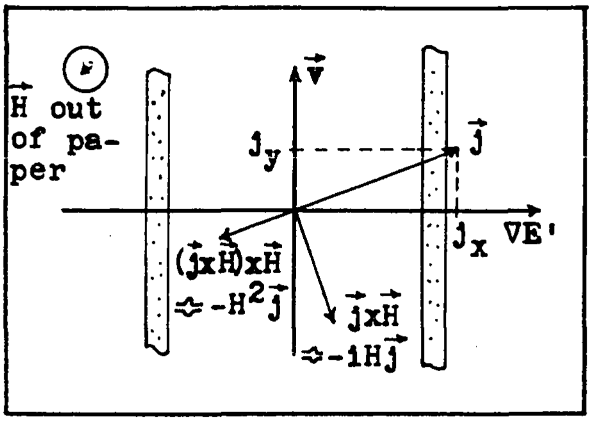


Fig. 4. Vector relationship in infinite slab geometry

Furthermore, when $f \approx 1$, $K = \nu/\omega \approx 0$, as electrons rarely encounter ions and A, C and D are given by:

$$A = \mu(K + K_-) \simeq \mu K_-$$
$$C = \mu \qquad 42$$
$$D = 0$$

Then, solving for the real part of $\vec{j}$, j_x:

$$j_x = \frac{n_- e \nabla E'}{H} \frac{\mu \kappa_-}{\mu^2 \kappa_-^2 + \mu^2} = \frac{n_- e \nabla E'}{\mu H} \frac{\nu/\omega}{1+(\nu/\omega)^2} \qquad 43$$

Finally, dividing both numerator and denominator by ν^2/ω^2 and replacing ω by $\mu He/m$ and ν by $n\sigma v_-$ (where, for the sake of convenience, the electron velocity $\vec{v} + \vec{V}_+ + \vec{V}$ is designated by $\vec{v}_-$) before solving for η_x or $j_x/\nabla E'$, will give the result:

$$\eta_x = j_x/\nabla E' = \frac{e^2}{mv_-} \frac{n_-}{n\sigma} \frac{1}{1+(\omega/\nu)^2} \qquad 44$$

This last equation shows that the conductivity in the x direction is reduced by the factor $1+(\omega/\nu)^2$ by the magnetic field.

It should be noted that when $\omega/\nu < 1$, i.e., when the cyclotron frequency is less than the collision frequency, the electrons cannot spiral freely between collisions, and, hence the effect on the conductivity is not very great. This corresponds to a weak magnetic field or a fairly dense gas. Conversely, when the gas is very rare or the magnetic field very strong, the conductivity transverse to the magnetic field may be drastically reduced, as the electrons can spiral freely, and hence spend much time orbiting around a line of force with a corresponding decrease in mobility.

Generally, most of the current is carried by the electrons, as these have a greater mobility than the ions. However, the situation may prevail where the electrons spiral freely but not the ions, and hence the latter would, in this case, be the main charge carriers. This could be the case in a rarefied gas in not too strong a magnetic field.

D. The Electron Scatter Cross Section

When a low energy electron is elastically scattered by an atom, the cross section is found to be energy dependent. This is because the de Broglie wave length of the electron is energy dependent and the scattering is due to the diffraction of the wave by the potential field of the atom. The energy dependence of the cross section is quite reminiscient of the energy variation in nuclear neutron cross sections.

The quantum mechanical treatment for computing the cross section was developed by Allis and Morse (20).

For any given element the cross section depends on the electron energy and the characteristics of the potential field surrounding the atom. For molecules the process is more complicated, one reason being that the electron may impart rotation to the molecule.

Massey (21) states, however, that highly symmetrical molecules, such as methane and various tetrahalides, may be treated as individual atoms. For example, when the potential distribution of the methane molecule is averaged to an even, spherically symmetrical distribution, a calculation of the cross section based on the one atom concept yields results which are in good agreement with experimental data (and, incidentally, corresponds very closely with the values computed similarly for an Argon atom).

The Schroedinger equation for an electron of kinetic energy E in a potential field V(r) is:

$$\nabla^2 \psi + \frac{8\pi^2 m}{h^2}(E-V(r))\psi = 0 \qquad 45$$

The equation may also be written as a function of the wave number, k, of the electron. Since $k^2 = 2mE/\hbar^2$, when $(8\pi^2 m/h^2)V(r)$ is further replaced by U(r), the equation takes on the form:

$$\nabla^2 \psi + (k^2-U(r))\psi = 0 \qquad 46$$

It can then be shown that the total elastic scattering cross section of the atom for the electron will be given by:

$$\sigma = \frac{4\pi}{k^2} \sum_q (2q+1)\sin^2 \theta_q \qquad 47$$

where θ_q represents the phase shift in the q^{th} mode of the diffracted wave. The value of θ_q depends on the form of V(r). However, in the case of very low energy electrons ($\lesssim 1$ e.v.) only θ_o is important. As $\sin^2 \theta_o$ can, at most, have a value of unity, then, in the case of very low energy electrons, the maximum value of the cross section can be computed approximately from the relation:

$$\sigma_{max} \approx \frac{4\pi}{k^2} \qquad 48$$

III. PRELIMINARY DESIGN OF THE GENERATING SYSTEM

A. Qualitative Design Considerations

The possibility of extracting electric power from a jet of hot gases can be demonstrated experimentally by a relatively simple means, such as the device illustrated in Fig. 5 (25). The question then arises about the feasibility of a system utilizing the same principles for large scale power production.

From Saha's equation it is seen that as high an operating temperature as possible is desirable to promote ionization. A high temperature is obviously also desirable from the thermodynamic point of view. As the temperature of chemical heat sources is generally limited to less than about 2000-3000^{o}K, a fission powered source would seem to be preferable. Furthermore, because of the high temperature, a fissile gas reactor seems the logical choice.

The construction material for such a reactor must have a low neutron absorption cross section, be capable of withstanding very high temperatures, and, if the reactor is to be thermal, have good moderating properties. This limits the choice in the latter case essentially to graphite.

Graphite has a sublimation point of approximately 4200^{o}K under a pressure of one atmosphere. It has the

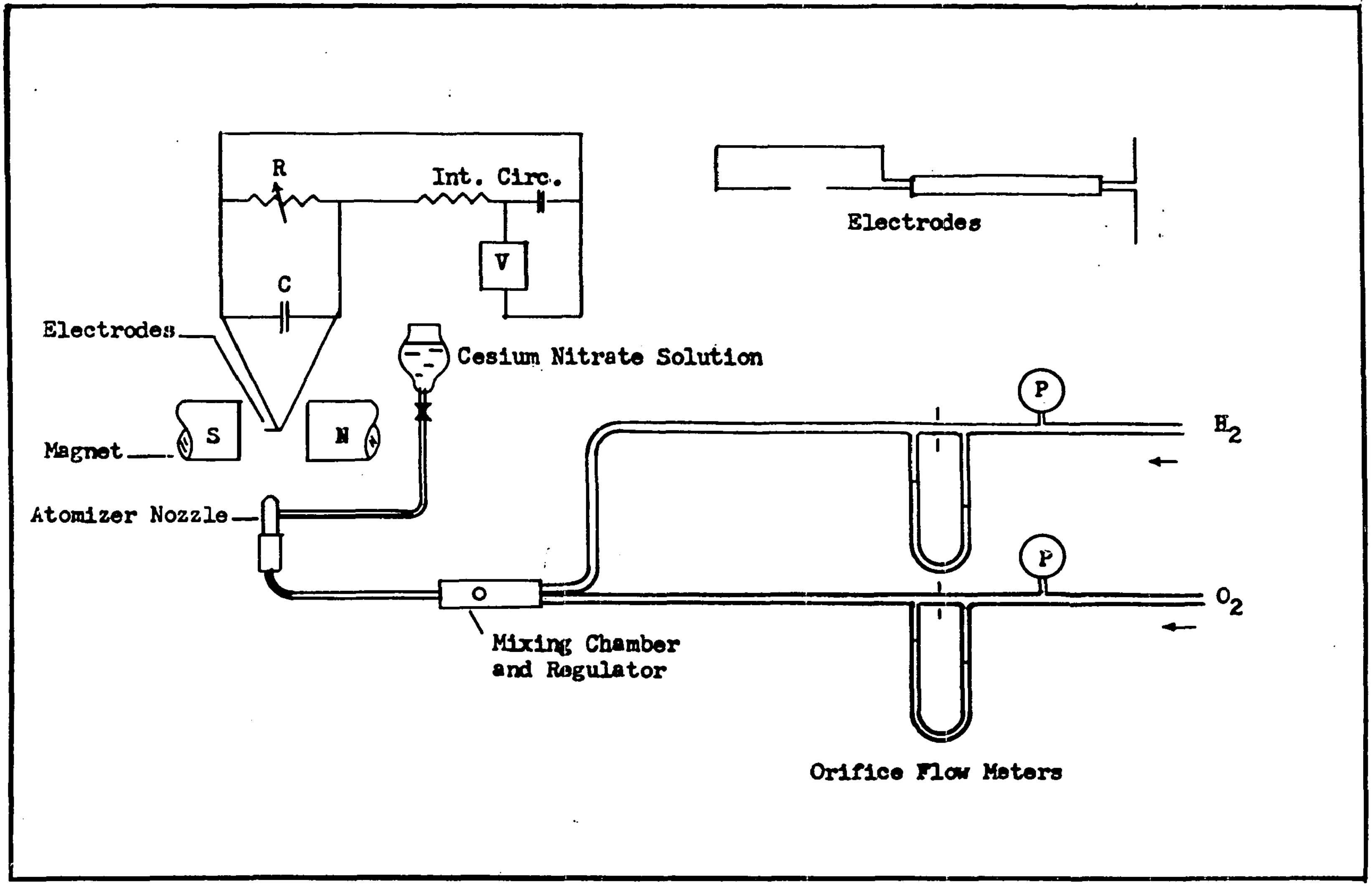


Fig. 5. Apparatus for demonstrating induced charge separation in a hydrogen-oxygen flame

somewhat unique property that its tensile strength actually increases up to about 3000°K where a maximum of 7000 p.s.i. is reached, whereafter the tensile strength again decreases rapidly up to the sublimation point (27). Consequently, 3000°K will be selected as the upper temperature limit in the system.

The fuel will have to be a U^{235} compound which is gaseous at 3000°K, will not dissociate (to form free uranium) at that temperature and must not react with graphite. The reason that chemical stability is required is that even at 3000°K the vapour pressure of uranium is still low enough so as to cause condensation of the liquid metal to take place in the system unless the partial pressure of free uranium vapour is very small. In the cooler parts of the system the free uranium might also react with the graphite to form liquid uranium carbide which would flow away from the regions where it would be formed and thus erode away the graphite. Furthermore, the gas itself should not react with graphite either. Another desirable characteristic should be a small neutron capture (as contrasted with fission) cross section of the fuel.

A compound which seems to fulfill all these criteria is enriched uranium tetrafluoride. Brewer, et al., (28) have tabulated some of the thermodynamic properties of uranium halides.

Under a pressure of one atmosphere uranium tetrafluoride is stable up to at least 3000°K. The fluorine atoms in the molecule will increase the neutron capture cross section but slightly, as fluorine occurs only as one isotope, F^{19}, with a cross section of 9 millibarns.

The lower limit of the heat sink temperature in the system will, of course, depend on the pressure. Fig. 6 shows the vapour pressure temperature relationship for uranium tetrafluoride.

The choice of uranium tetrafluoride poses one problem, however, for the very reason that it is so stable at high temperatures it is likely to have a high ionization potential. Fluorine itself has an ionization potential of 17.34 v. In order to promote ion formation, therefore, the uranium tetrafluoride must be laced with some other gas that has a low ionization potential. The additive must, in addition to not reacting with the graphite, also be chemically stable in the presence of the fuel.

The logical choice would seem to be cesium fluoride. At 3000°K it will dissociate to some extent into free cesium and fluorine gases. The cesium which has the lowest ionization potential of any element will ionize to a slight extent, thus increasing the conductivity of the mixture.

The cesium will not react with the uranium tetrafluoride gas as long as there is free fluorine present,

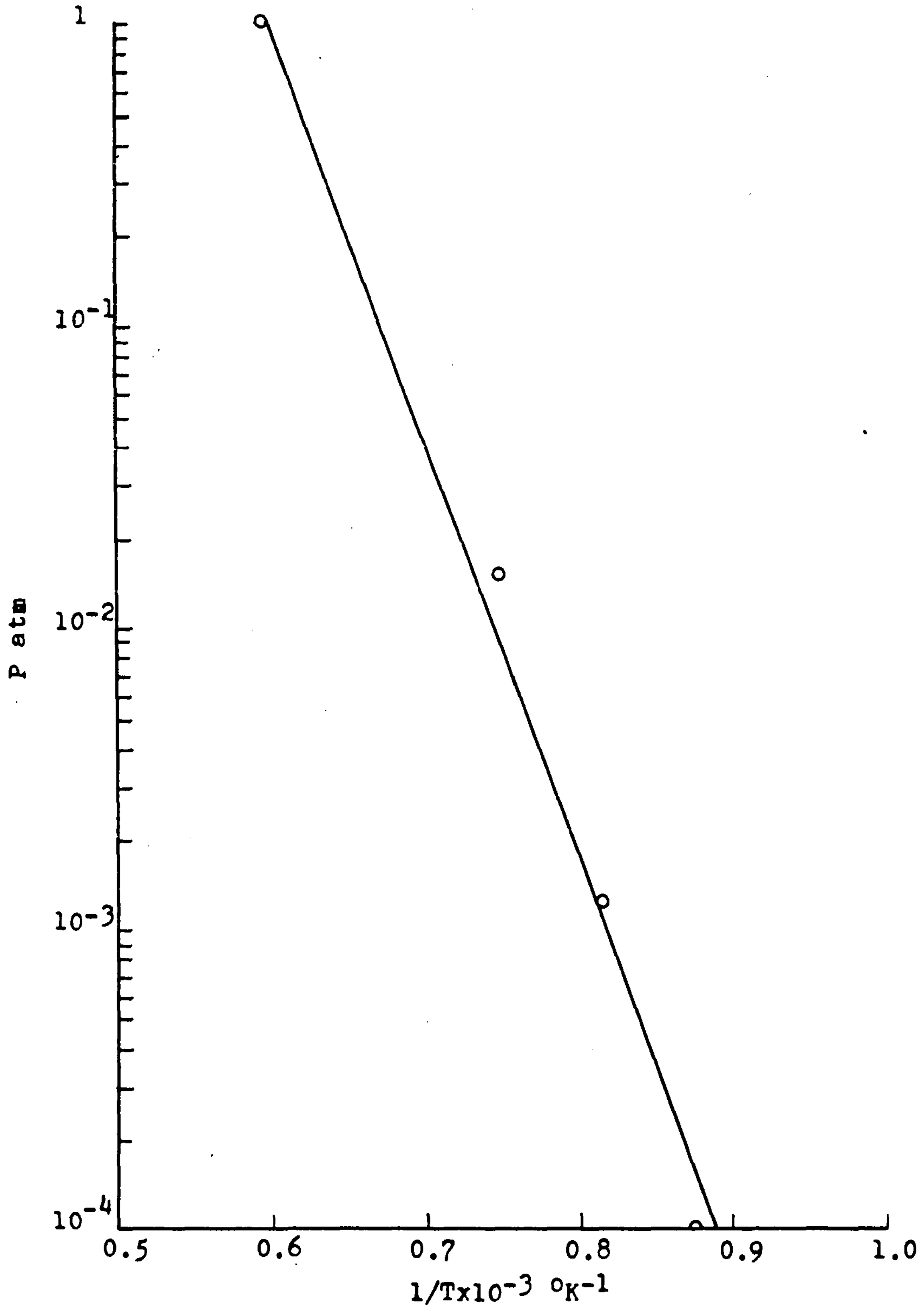


Fig. 6. Vapour pressure of uranium tetrafluoride

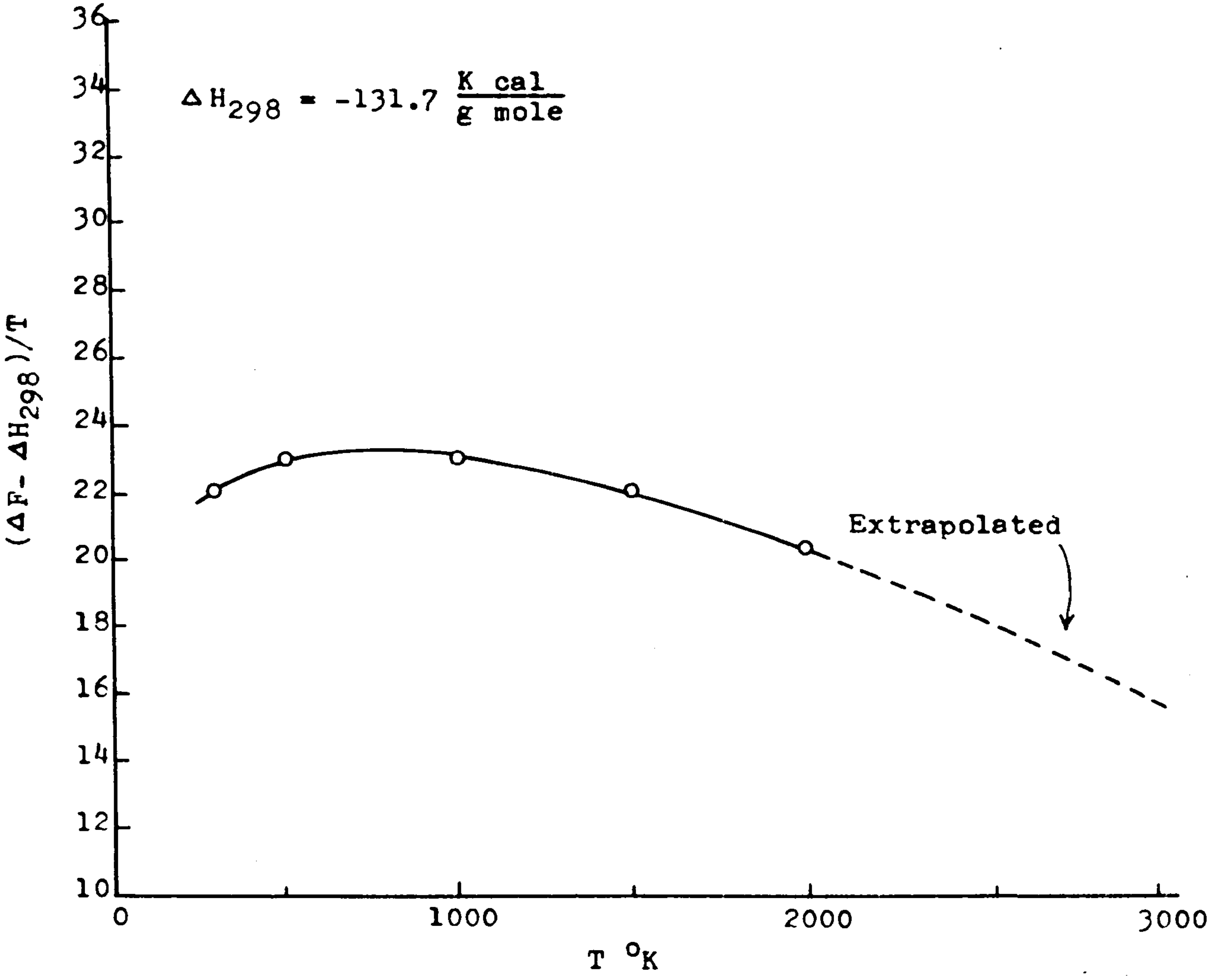


Fig. 7. Free energy function for cesium fluoride

with which it will react preferentially. Nor will the fluorine react with the graphite, as carbon tetrafluoride is completely unstable at the operating temperature. The degree of dissociation of the cesium fluoride may be calculated from the free energy of formation of cesium fluoride gas. Quill (29) gives the free energy function $(\Delta F-\Delta H_{298})/T$ at various temperatures. This data is reproduced in Fig. 7.

It is fortunate that cesium also has a very low neutron capture cross section, for if the contrary were true, all its other advantages as an additive to the fuel might be nullified by its poisoning of the fission reaction. Cesium consists of but one isotope, Cs^{133}, with an absorption cross section of 17 m.b.

The gas velocity of the uranium tetrafluoride in the reactor would probably not be great enough to render mechanical erosion of the graphite a problem. However, at high temperatures graphite is quite permeable to many gases, although it is debatable whether the uranium tetrafluoride molecule might not be large enough to be an exception. Should the uranium tetrafluoride prove to diffuse through graphite at high temperatures it might prove necessary to coat the gas ducts with pyrographite, which is reported to be much more impermeable.

The hot gas from the reactor would next be expanded through a nozzle into a generating section as is schematically indicated in Fig. 8.

An electrical insulator will be required to insulate the graphite from the electrical generating section as the graphite is a conductor and might otherwise short out the potential generated in this section. Again pyrographite looks like a promising prospect. This is because it is very anisotropic with respect to electrical conductivity having a very slight electrical conductivity in the direction crosswise to its molecular layers as contrasted with its greater (by a factor of 1000) conductivity in a direction parallel to the molecular planes. When oriented accordingly, it can serve as an insulator, a conductor or both. Alternatively, one could use a ceramic material capable of withstanding high temperatures, such as niobium carbide, which has a melting point of about 3773°K.

In the scheme presented in Fig. 8 the divergent portion of the nozzle is simultaneously used to serve the function of electrodes in the generating section. The gas will be accelerated to a supersonic speed and, upon entering the magnetic field region, a current will be induced in it. If the divergent portion of the nozzle could be constructed from a single section of pyrographite (which is not feasible at present), then the pyrographite could also be used to electrically connect the gas to an external load. This

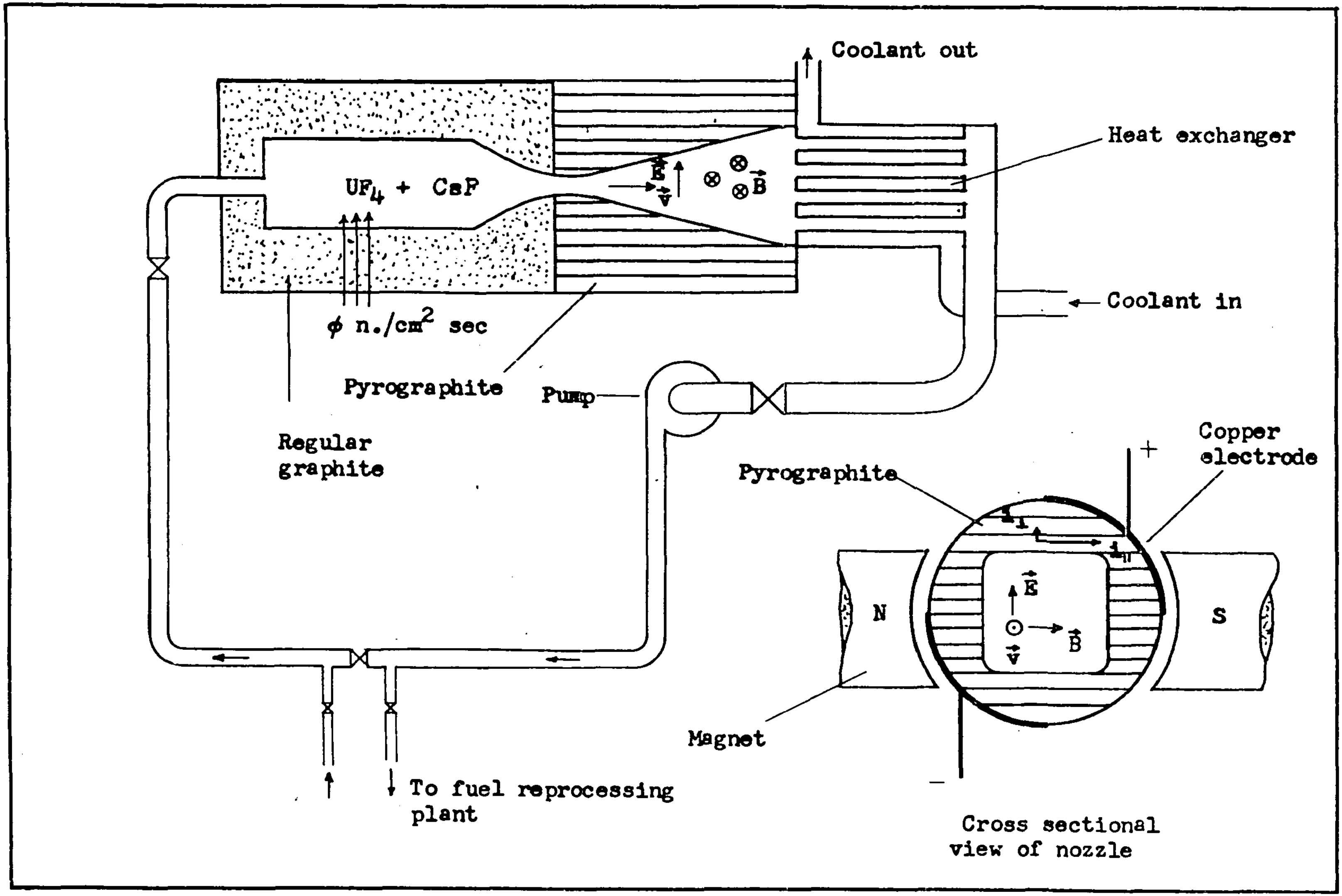


Fig. 8. A schematic diagram of a postulated device for power generation from a gas thermally ionized by fission

could be accomplished by connecting the external load to the nozzle in such a manner that the current flow to the load would be in the direction parallel to the molecular layers within the graphite. At the same time it would not short out the potential established across the gas jet in the direction perpendicular to the planes of molecular stratification, because of its acting as an insulator in that direction. However, this is all speculative. A more down to earth solution would be to incorporate in the walls of the nozzle two graphite electrodes electrically insulated from one another, excepting the connection through the external load. There are some serious doubts about the ability of graphite to withstand erosion by hot gases at the high velocities that would prevail in the nozzle. Pyrographite would be more resistant.

Barring both of these, though, it might be necessary to contemplate using some other conductor capable of withstanding high temperatures. Tungsten might be a possible material. Rom (23) suggests that tungsten may be used for structural purposes at temperatures up to about 3000°K, at which it has a tensile strength comparable to graphite at the same temperature. The trouble with tungsten is that it has a large neutron absorption cross section. If it were to be eroded from the generating section and would carry over into the reactor section it might cause a bothersome reduction in the reactivity.

For any appreciable field strength the magnetic field would have to be formed by an electromagnet. With conventional copper coils a field of the order of 10 kilogauss might be feasible. A more promising approach might be to use superconductive coils constructed of a niobium-tin alloy that becomes superconductive at 18^{o}K and retains its superconductivity in magnetic fields up to 100 kilogauss in strength. This would require liquid helium cooling of the coils but would use up no electrical energy as ohmic losses therein.

The gas will be slowed down by the field when it carries a current, i.e., when energy is extracted from it. Thermodynamically this is equivalent to shaft work in a turbine. It is interesting to note that the energy transfer mechanism differs somewhat from that applying to a solid conductor moving in a magnetic field. In the case of an ionized gas which is not at a very low pressure, the magnetic field exerts a force and a braking action primarily on the electrons; the action is electrostatically transmitted to the ions and these produce a resultant drag on the neutral gas by collisions with its molecules.

If the gas were slowed down to zero velocity this would correspond to no potential drop across the gas, i.e., to a shorted electromotive cell. Conversely, if the electrodes are charged to the maximum possible potential, this corresponds to the open circuit voltage of the cell and no current will be drawn. Therefore, the gas cannot be slowed down to stagnation if power is to be drawn from the device, provided each electrode is electrically continuous, i.e., at a uniform potential over the whole length of the slowing down region. It has not been investigated whether greater efficiency could be obtained were the electrodes to be segmented, with each subsequent segment pair series connected to the previous one.

Upon leaving the generating section the gas, now cooled by expansion, will enter a heat exchanger where it will be cooled further, prior to being recirculated through the cycle. This is necessary, because, in order to get net work out of the cycle, the system must include a heat sink. The material used in the heat exchanger will most probably have to be tungsten, although, depending on the temperature of the gas at this stage, there might be other alternatives. The same will be true for the other parts of the system that are external to the reactor, generator and heat exchanger sections and in contact with the gas. The cooled low pressure gas will then have to be recompressed prior to returning it to the reactor. In order to replenish the U^{235} consumed and to remove fission products, a small amount of the gas must be constantly bled off for processing.

From a thermodynamic point of view the system would work on a Breyton cycle, the generating section taking the place of the turbine.

A system such as the one schematically represented in Fig. 8 would, of course, not be limited to one kind of reactor. One can conceive that the reactor could be either fast or thermal. If thermal, it could be either homogenous, for example, by mixing the fissile gas with deuterium, or it could be made heterogenous by allowing the gas to flow through a graphite matrix. Finally, the fission chamber, per se, might be subcritical and supplied with a thermal neutron flux by an external reactor. For example, it might be situated in the center of an annular reactor.

Furthermore, the working fluid itself need perhaps not be fissile; it might, for instance, be possible to let it carry a solid fuel such as uranium dioxide in a very fine suspension. It is also conceivable to use higher temperatures still, causing the reactor gas to form a plasma and magnetically confining it and expanding it through a magnetic nozzle into the generating section. In the following section the main design aspects for a 1000 MW heterogeneous thermal reactor will be contemplated.

B. Calculation of Values of Main Reactor Design Parameters

Thermodynamic cycle

As stated previously, the whole system would operate on a Breyton cycle. The first step in the design is to fix the operating conditions and determine the cycle.

The upper temperature limit for the gas emerging from the reactor has already been set at 3000°K. It is advisable to operate at as high a pressure as possible, in order to obtain a high energy density. For equipment having a size similar to that of such a reactor a pressure of 100 atm. is about the practical upper limit. As will be seen later there are also electrical considerations for selecting such a high pressure.

Brewer, et al. (28), give data enabling the entropy of saturated uranium tetrafluoride vapour at 1 atm. to be calculated.

Now, the value of the molar specific heat at constant volume, C_v, for the gas may be estimated if one assumes ideal gas behavior, which seems permissible at the temperatures involved. For a gas having nonlinear polyatomic molecules of n atoms, kinetic theory predicts a constant volume specific heat value of (3n-3)R, where R is the gas constant of 8.32 joules/g mole deg K. Thus, for UF_4:

$$C_v = (3n-3)R = (3x5-3)x8.32$$
$$= 99.68 \text{ joules/g mole deg K} \qquad 49$$

Then the corresponding value of the specific heat at constant pressure, C_p, may be calculated from the relation:

$$C_p = C_v + R = 99.68 + 8.32$$
$$= 108 \text{ joules/g mole deg K} \qquad 50$$

Hence, the ratio of the specific heats is found to be, $\gamma = C_p/C_v = 108/99.62 = 1.082$ and $(\gamma-1)/\gamma$ consequently will be 0.082/1.082 = 0.0758. This, when taken together with the known entropy of saturated UF_4 vapour at 1 atm., makes it possible to calculate the entropy of the gas at various temperatures and pressures by using the familiar relation:

$$\Delta S = C_p \ln \frac{T_2}{T_1} = 108 \ln \frac{T_2}{T_1} \qquad 51$$

and,

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = T_1\left(\frac{P_2}{P_1}\right)^{0.0758} \qquad 52$$

When the computed values are considered along with the vapour pressure data, a TS diagram for the gas may be constructed. This has been done, and the diagram is shown in Fig. 9.

It should be noted that, because of the polyatomic nature of the UF_4 molecule and the resulting high heat capacity, the exponent on the pressure ratio in equation 52 is small. This means that the gas can undergo a considerable adiabatic pressure drop with only a relatively small drop in temperature.

Because the thermodynamic efficiency of the device is enhanced by a low heat sink temperature, it is preferable to expand the gas leaving the reactor to as low a pressure as feasible. In rocket motors, the expansion ratio is generally limited to a value of about thirty. This is because nozzles tend to become excessively large if the expansion is carried out beyond this point. Therefore, the lower limit of the pressure in the system will be taken as 3 atm., corresponding roughly to a thirtyfold expansion. From the TS diagram this is seen to correspond to a temperature of 2260°K.

Ohmic heating in the generator will reheat the gas to some extent, causing a pressure rise to 12.3 atm. The gas will then be cooled at a constant pressure to 2000°K (in the ideal case), then adiabatically recompressed until at 2320°K, whereupon it will be returned to the reactor.

The unweighted mean of the reactor entrance and exit

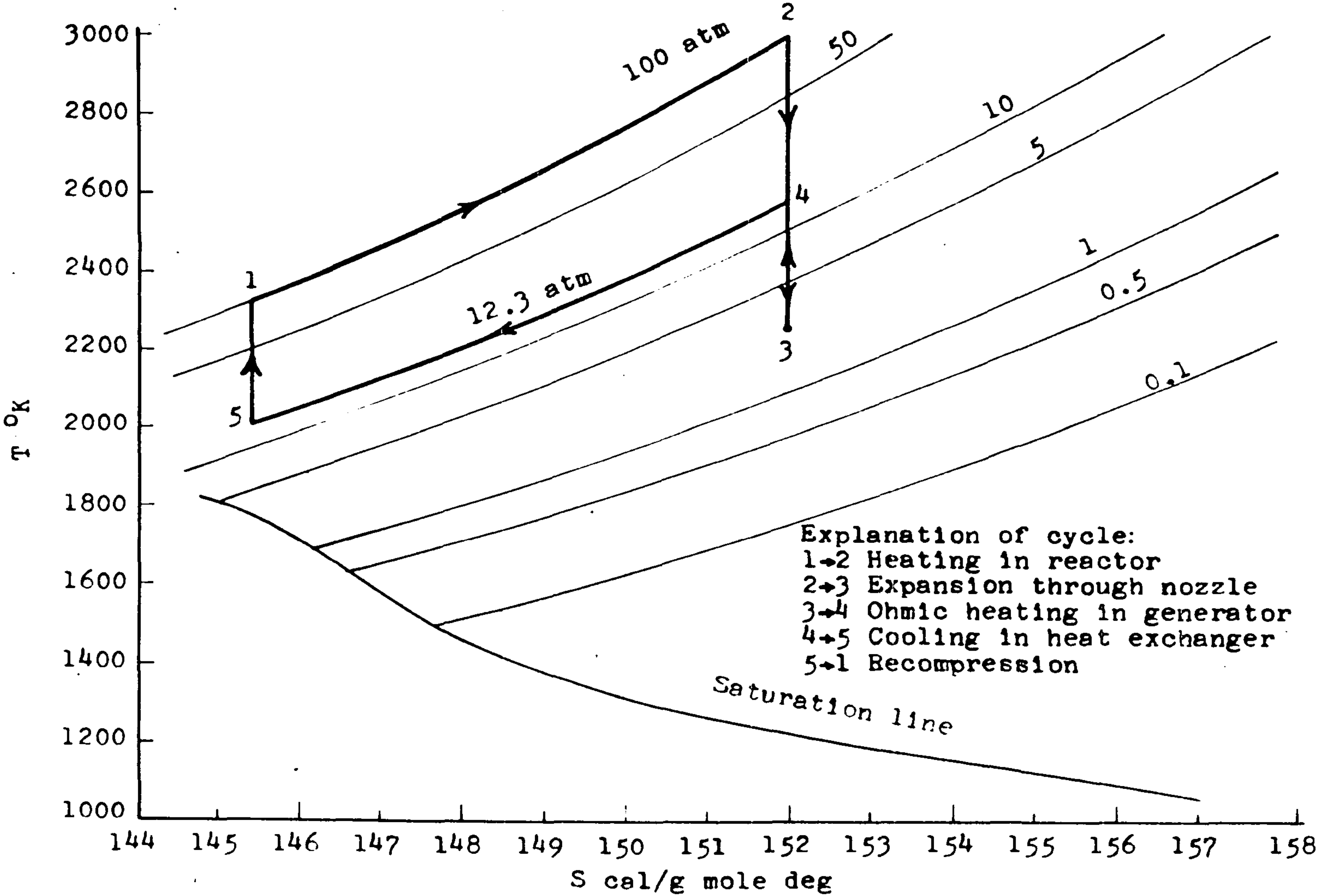


Fig. 9. TS diagram for uranium tetrafluoride and proposed thermodynamic cycle

temperatures is 2660°K. However, because the density of the gas is greater at the lower temperatures encountered in the reactor, there will be a greater mass of gas at temperatures close to the entry temperature than at those close to the exit temperature. Thus, the weighted average will be pulled down from the value of 2660°K, and will lie somewhere between that value and 2320°K, probably closer to the higher value. The weighted mean value will probably be some 200 degrees higher than the entrance temperature of the gas and this will be assumed. This is a pure assumption, but is at least accurate to within ten percent, more probably within five. Therefore, 2520°K will be taken as the mean temperature within the reactor.

Fuel inventory and neutron flux

The temperature rise of the gas in the core is 3000-2320 = 680 degrees. As the thermal power level, Q, of the reactor is to be 1000 MW or 10^9 watts, this means that the gas removal rate, q, must be:

$$q = \frac{Q}{C_p \Delta T} = \frac{10^9}{1.08\times10^2 \times 6.8\times10^2} = 1.36\times10^4 \text{ g moles/sec} \qquad 53$$

Now, the molal density may be calculated from the ideal gas equation:

$$\rho_M = \frac{P}{RT} = \frac{100}{0.082x2520} = 0.485 \text{ g moles/1} \qquad 54$$

or 485 g moles/m^3. Then, using the continuity equation, the total cross sectional flow area, s, may be computed.

Taking the maximum feasible average gas velocity in the core, v, to be 100 ft/sec or 30.5 m/sec:

$$s = q/v\rho_M = \frac{1.36x10^4}{30.5x485} = 0.92 \text{ m}^2 \qquad 55$$

For a high power reactor, such as the one under consideration, the minimum fuel inventory will be determined by the maximum permissible flux, rather than by critical size considerations.

As there are only graphite and UF_4 present in the core, the neutron flux level can be quite high, as radiation damage will not be as pertinent a factor as when other structural materials, in addition to solid fuel, are used.

For a 1000 MW thermal power level there must be $3.1x10^{19}$ fissions/sec. The effective fission cross section of U^{235} at the reactor temperature will be:

$$\sigma_f = \sigma_{f,293} f \frac{\sqrt{\pi}}{2} \left(\frac{293}{T}\right)^{\frac{1}{2}}$$

$$= 580 \times 0.98 \frac{\sqrt{\pi}}{2} \left(\frac{293}{2520}\right)^{\frac{1}{2}} = 170 \text{ b} \qquad 56$$

where f is the non-1/v factor.

Tentatively, assuming that fully enriched UF_4 gas will be used, let the average flux in the core be 6.5×10^{14} n/cm^2 sec. This would correspond to a peak flux of about 10^{15} n/cm^2 sec in a well reflected cylindrical reactor of optimum dimensions with uniformly distributed fuel. This, of course, differs considerably from the case under consideration, but provides a preliminary reference point if one proceeds on the basis of limiting the peak flux to a value comparable to 10^{15} n/cm^2 sec. Then the quantity of fuel in the reactor core may be determined from the fission rate equation:

$$-\frac{dN}{dt} = N\sigma_f \phi \qquad 57$$

therefore,

$$N = -\frac{dN}{dt}\frac{1}{\sigma_f \phi} = \frac{3.1 \times 10^{19}}{170 \times 10^{-24} \times 6.5 \times 10^{14}}$$

$$= 2.81 \times 10^{26} \text{ atoms} \qquad 58$$

The fuel inventory will thus be $2.81\times10^{26}/6.03\times10^{23} = 467$ g atoms U^{235} or $467\times235 = 109{,}500$ grams.

Core

The core will essentially consist of a cylindrical block of graphite with longitudinal gas passagos running through it, arranged in a hexagonal manner.

In order to accommodate 467 g moles of UF_4 at a molal density of 485 g moles/m^3, the volume of the gas must be $467/485 = 0.964$ m^3. As the cross section of the gas ducts is 0.92 m^2 the height, h, will be $0.964/0.92 = 1.045$ m.

At 2520°K the atomic density of graphite, $\bar{N}_m$, will be less than that at 293°K by a factor of $(1+(2520-293)\beta)$, where β is the volume expansion ratio of 2.36×10^{-5} deg^{-1}. This factor will then equal $1+2.27\times10^3\times2.36\times10^{-5} = 1.0525$. Then $\bar{N}_m$ may be calculated:

$$\bar{N}_m = \frac{\rho_{m,293}}{A_m}\,\frac{N_{Av}}{1+\beta\Delta T} = \frac{1.6}{12}\,\frac{6.03\times10^{23}}{1.053}$$

$$= 7.62\times10^{22} \text{ atoms/cm} \qquad 59$$

where ρ_m refers to the density of graphite and A_m its atomic weight. The atomic density of the fuel, $\bar{N}_U$, will be $2.81\times10^{26}/9.64\times10^5 = 2.92\times10^{20}$ atoms/cm^3. The atomic ratio of moderator to fuel in the core will be $7.62\times10^{22}/2.92\times10^{20} = 251$ for equal volumes of the two materials.

A moderator to fuel atomic ratio of 500 will require a critical mass of 80 kg for a bare homogeneous spherical reactor. (See Glasstone (30) p. 188) Even if the core under consideration will be cylindrical, heterogeneous, and of non optimum dimensions (i.e., height to radius ratio of less than 1.845) it will be reflected, and thus, a moderator to fuel ratio of 500 should suffice for criticality with a fuel mass of 110 kg.

Another reason for selecting a higher moderator to fuel atomic ratio than may be strictly necessary for criticality is for mechanical reasons, namely, if there is too little graphite in the core, the spacing between the ducts will be insufficient for a good mechanical strength of the graphite matrix.

Because the core height is the same as the gas duct length, the volume ratio of moderator to fuel will be the same as the ratio of the cross sectional area of the graphite to the total cross sectional area, s, of the fuel ducts. Then, settling for a moderator to fuel atomic ratio of 502, or a corresponding volume ratio of two to one, this will mean that the cross sectional area of the graphite will be 2x0.92 = 1.84 m^2 and that of the whole core 3x0.92 = 2.76 m^2. This corresponds to a total core diameter of 1.88 m. The core height, it will be remembered, is 1.045 m.

Unit cell

After the atomic ratio of moderator to fuel has been decided on, the optimum size of the unit cell may be determined.

A hexagonal lattice is preferable because it will give the most even distribution of graphite around the gas ducts. Such a lattice will require the number of cells to be 7, 19, 37 or 61 (or greater). It is preferable to have the number of cells as small as possible, i.e., to make the unit cell as large as possible, as this entails large gas ducts, and hence minimizes friction losses in the gas.

However, as the size of the unit cell is increased, the thermal utilization factor, f, is diminished. The minimum number of cells is thus determined by how low the value of f may be. Seven cells were thus found to be too few. Consequently, taking the next number in the series, 19, to be the number of cells, the cross sectional area of the gas duct within the cell will be the total cross sectional area available for gas flow divided by the cell number, or $0.92/19 = 0.0483\ m^2$ or $483\ cm^2$, corresponding to a duct radius, r_0, of 12.4 cm.

The ratio of the cross sectional area of the graphite to that of the fuel duct will, of course, be the same as for the core as a whole, namely, two to one. Geometric considerations will show that when the graphite to duct cross sectional area ratio has this value the width of the cell will be $2\sqrt{(\sqrt{3}/2)\pi}\, r_o = 3.24r_o$, and the side of the cell $\sqrt{(2/\sqrt{3})\pi}\ r_o = 1.905\ r_o$. Hence, for a duct radius of 12.4 cm the cell width will be 3.24x12.4 = 37.8 cm and its side 1.905x12.4 = 23.6 cm. The cross section of the unit cell is shown in Fig. 10.

Now that the cell dimensions have been determined, the effect on the thermal utilization may be calculated. The radius of an equivalent cylindrical cell, r_1, will be $\sqrt{3}\ r_o = 1.73r_o = 1.73\text{x}12.4 = 21.4$ cm, because the cross sectional area of the cell is three times that of the fuel duct. Again, taking 2520°K as the mean reactor temperature, the diffusion length, L, of the graphite may be calculated.

Referring back to p. 48 the volume expansion of the graphite was found to be 5.2%, hence this value will also represent the decrease in its density. Then, using the equation:

$$L^2_T = L^2_{298} \left(\frac{\rho_{298}}{\rho_T}\right)^2 \left(\frac{T}{298}\right)^{0.6}$$

$$= 2525 \text{x} 1.052^2 \text{x} \left(\frac{2520}{298}\right)^{0.6} = 10{,}100 \text{ cm}^2 \qquad 60$$

therefore,

$$\kappa_1 = 1/L_{2520} = 1/(10{,}100)^{\frac{1}{2}} \text{cm}$$

$$= 9.9 \text{x} 10^{-3} \text{cm}^{-1} \qquad 61$$

Now, for the fuel, transport theory must be used in calculating κ_0. Taking the microscopic scattering cross section of the U^{235} nucleus at 2520°K as 6.5 b, and the absorption to fission cross section ratio as 1.18, further remembering that the fission cross section had been calculated to be 170 b and the atomic density $2.92 \text{x} 10^{20}$ atoms/cm^3, then the macroscopic total cross section of the fuel, Σ, will be:

$$\Sigma = \bar{N}_U (1.18 \text{x} \sigma_f + \sigma_s)$$

$$= 2.92 \text{x} 10^{20} (170 \text{x} 10^{-24} \text{x} 1.18 + 6.5 \text{x} 10^{-24})$$

$$= 6.04 \text{x} 10^{-2} \text{ cm}^{-1} \qquad 62$$

The macroscopic scattering cross section Σ_s, of the fuel will, proportionately, be:

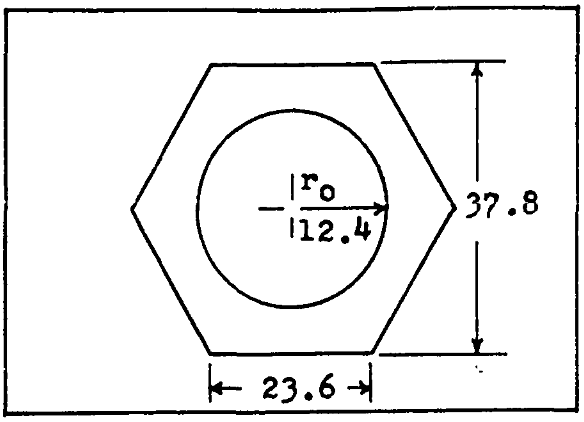


Fig. 10. Cross section of unit cell

$$\Sigma_s = \Sigma \frac{\sigma_s}{1.18\,\sigma_f + \sigma_s} = 6.04\text{x}10^{-2}\,\frac{6.5}{1.18\text{x}170+6.5}$$

$$= 1.9\text{x}10^{-3}\text{cm}^{-1} \qquad 63$$

Then, using the transport theory relationship $\kappa_o/\Sigma = \tanh(\kappa_o/\Sigma_s)$:

$$\frac{\kappa_o}{6.04\text{x}10^{-2}} = \tanh\frac{\kappa_o}{1.9\text{x}10^{-3}} \qquad 64$$

This equation is found to have a solution, $\kappa_o \approx \Sigma = 6.04\times10^{-2}$. This is easily seen as Σ_s is quite small, hence tanh $\kappa_o/\Sigma_s \approx 1$.

Then,

$$\kappa_o r_o = 6.04\times10^{-2}\times12.4 = 0.749$$

$$\kappa_1 r_o = 9.9\times10^{-3}\times12.4 = 0.123 \qquad 65$$

$$\kappa_1 r_1 = 9.9\times10^{-3}\times21.4 = 0.212$$

The Bessel functions required for the calculation of the thermal utilization and their values are:

$$K_o(\kappa_1 r_o) = K_o(0.123) = 2.224$$

$$K_o(\kappa_1 r_o) = K_1(0.123) = 7.973$$

$$K_o(\kappa_1 r_1(= K_1(0.212) = 4.487$$

$$I_o(\kappa_1 r_o) = I_o(0.123) = 1.0038$$

$$I_1(\kappa_1 r_o) = I_1(0.123) = 0.0616$$

$$I_1(\kappa_1 r_1) = I_1(0.212) = 0.1066$$

$$I_o(\kappa_o r_o) = I_o(0.749) = 1.1452$$

$$I_1(\kappa_o r_o) = I_1(0.749) = 0.4014$$

then,

$$M_o(\kappa_1 r_o) = K_o(\kappa_1 r_o) + \frac{K_1(\kappa_1 r_1)}{I_1(\kappa_1 r_1)} I_o(\kappa_1 r_o)$$

$$= 2.224 + \frac{4.487}{0.1066} 1.0038 = 44.57 \qquad 67$$

and,

$$M_1(\kappa_1 r_o) = K_1(\kappa_1 r_o) - \frac{K_1(\kappa_1 r_1)}{I_1(\kappa_1 r_1)} I_1(\kappa_1 r_o)$$

$$= 7.973 - \frac{4.487}{0.1066} 0.0616 = 5.373 \qquad 68$$

Further, defining F and E-1,

$$F = \frac{\kappa_o r_o}{2} \frac{I_o(\kappa_o r_o)}{I_1(\kappa_o r_o)} = \frac{0.749}{2} \frac{1.1452}{0.4014} = 1.068 \qquad 69$$

$$E-1 = \frac{V_1}{V_o} \frac{\kappa_1 r_o}{2} \frac{M_o(\kappa_1 r_o)}{M_1(\kappa_1 r_o)} - 1$$

$$= 2 \frac{0.123}{2} \frac{44.57}{5.373} - 1 = 0.023 \qquad 70$$

Now, the absorption cross section of the fuel, Σ_o, will be $\Sigma - \Sigma_s = 6.04 \times 10^{-2} - 1.9 \times 10^{-3} = 5.85 \times 10^{-2} cm^{-1}$. The macroscopic absorption cross section of the moderator can also be calculated. The atomic density of the moderator was previously computed to be 7.62×10^{22} atoms/cm^3. The microscopic absorption cross section at room temperature is 3.2 millibarns. Hence, making the appropriate temperature corrections, at 2520°K the macroscopic cross section, Σ_1, will be:

$$\Sigma_1 = \bar{N}_m \sigma_{a,293} \frac{\sqrt{\pi}}{2} \left(\frac{293}{T}\right)^{\frac{1}{2}}$$

$$= 7.62\text{x}10^{22}\text{x}3.2\text{x}10^{-27}\text{x}1.128^{-1}\left(\frac{293}{2520}\right)^{\frac{1}{2}}$$

$$= 7.36\text{x}10^{-5}\ \text{cm}^{-1} \qquad 71$$

Finally,

$$\frac{1}{f} = 1 + \frac{V_1}{V_0}\frac{\Sigma_1}{\Sigma_0} F + E - 1$$

$$= 1+2\ \frac{7.36\text{x}10^{-5}}{5.85\text{x}10^{-2}}\ 1.068+0.023 = 1.0256 \qquad 72$$

Therefore,

$$f = 1/1.0256 = 0.976 \qquad 73$$

Geometric buckling

It should be stressed that, because in the reactor under discussion the fuel is continuously being mixed, the first fundamental reactor theorem is not strictly applicable. This is a result of the delayed neutron emitters being continuously redistributed throughout the core (See Weinberg and Wigner (31) p. 599).

Furthermore, because of the nonuniform fuel concentration due to the density variation of the gas within the reactor, the neutron production rate may vary with distance along the core axis (although nonuniform fuel loading need not necessarily entail a spatially varying source term in the diffusion equation, as the fuel may be made to vary in a manner that compensates for flux variation).

The conventional method for solving the wave equation for a cylindrical geometry presupposes a symmetrical flux distribution with respect to the midplane perpendicular to the axis of the cylinder. This will be the result of a spatially independent source term in the diffusion equation. In the case under consideration, this may not be so and the flux may therefore be asymmetrical.

However, because the migration length is an appreciable fraction of the reactor height (and for lack of any better way to attack the problem), it seems reasonable to assume that the neutrons are fairly free to diffuse throughout the core prior to absorption, and thus tend to a symmetrical distribution.

Therefore, in spite of these two shortcomings, namely that due to the redistribution of delayed neutrons and, that due to the possible asymmetry of the flux, calculation of the geometric buckling will be based on the conventional approach. Should the result be in error in such a way as to give too high an estimate of the reactivity of the system, this will be compensated for by the fact that in the space above the core, in the convergent section of the nozzle, there will be an additional amount of fuel which will not be included in reactivity calculations. The effect of this fuel will be to increase the actual reactivity above the computed value.

It is proposed to have the core reflected on the sides by one layer of hexagonal graphite parallelepipeds of the same external dimensions as the unit cell. Thus the core, which has an average diameter of 1.88 m, will have essentially 18 more ductless unit cells added on to it to give a total of 37. The cross sectional area of the core plus reflector will thus be 37/19 times the cross sectional area of the core alone, which is 2.76 m^2. The total area will be (37/19)x2.76 = 5.38 m^2, corresponding to an average diameter of 2.62 m. Average diameters must be used because the core and reflector peripheries are not smooth surfaces as a result of the hexagonal shape of the unit cells of which they are composed. Hence, the average reflector thickness will be (262-188)/2 = 37 cm. It is also proposed to reflect the core on the bottom with the same thickness of graphite.

The reflector savings of the core, δ , are given approximately by the equation $\delta = L_r \tanh (t/L_r)$, where t is the reflector thickness and the subscript r refers to the reflector, which, in this case, is the same as the moderator. Thus:

$$\delta = 101 \tanh \frac{37}{101} = 35.4 \text{ cm} \qquad 74$$

The core will then be seen to be equivalent to an unreflected core having a diameter of 188+2x35.4 = 258.8 cm, or a radius, R, of 129.4 cm. The corresponding height will have one reflector savings value added to the core height to give an unreflected height of 104.5+35.4 = 139.9 cm.

The geometric buckling may now be calculated:

$$B^2_g = \left(\frac{2.405}{R}\right)^2 + \left(\frac{\pi}{h}\right)^2$$

$$= \left(\frac{2.405}{129.4}\right)^2 + \left(\frac{\pi}{139.9}\right)^2 = 8.50\text{x}10^{-4} \text{ cm}^{-2} \qquad 75$$

<u>Reactivity and material buckling</u>

As it is not practical to use fully enriched fuel in the reactor, the enrichment of the fuel must be decided on. The price per gram of highly enriched fuel does not increase substantially in the range from twenty to ninety percent. A high enrichment is desirable because of flux considerations. Eighty-five percent enriched fuel will suffice to keep the flux level reasonable, yet it can easily be maintained by ninety percent enriched makeup gas. Furthermore, eighty-five percent enriched fuel has an essentially undiminished η value. (the number of neutrons released by fission per thermal neutron captured in the fuel) Therefore, this value will be selected. This will raise the actual average flux level from the previously computed value of 6.5×10^{14} n/cm^2 sec, corresponding to fully enriched fuel, to $0.85^{-1} \times 6.5 \times 10^{14} = 7.53 \times 10^{14}$ n/cm^2 sec.

Because of the high enrichment and low density of the fuel, both the fast fission factor and the resonance escape probability will, for all intents and purposes, have a value of unity. Therefore, the infinite multiplication factor, k_∞, will be given simply by ηf or, when based on the previously used value of f of 0.976, by $2.07 \times 0.976 = 2.02$.

Actually, because the fuel density varies, the infinite multiplication factor will vary and hence the material buckling. However, proceeding on the same basis as was discussed in connection with the geometric buckling, an effective material buckling will be considered that corresponding to the previously mentioned value of f. Since the size of the reactor has already been decided on, there is no need to compute the actual value of the material buckling as long as it is greater than that of the geometric buckling, which is the case if the effective multiplication factor k_{eff} is larger than unity.

The Fermi age, τ, of the moderator, which is 350 cm^2 at room temperature, must be corrected to the temperature of the reactor. At room temperature the macroscopic scattering cross section of graphite, which has essentially the same value as the transport cross section, Σ_t, is 0.385 cm^{-1}. The mean logarithmic neutron energy decrement per collision, ξ, is 0.158. Hence, making both nuclear and density temperature corrections:

$$\tau = \left(\tau_{293} - \left[\frac{1}{3\xi\Sigma_s\Sigma_t}\right]_{293} \ln\frac{T}{293}\right)\left(\frac{\rho_{293}}{\rho_T}\right)^2$$

$$= \left(350 - \frac{1}{3\mathrm{x}0.158\mathrm{x}0.385^2}\ln\frac{2520}{293}\right) 1.11$$

$$= 354 \text{ cm}^2 \qquad 76$$

where the value 1.11 represents the square of the ratio of the densities previously computed. It is interesting to note that the Fermi age does not change much in going from room temperature to 2520°K. This is because the nuclear and density temperature coefficients tend to compensate for one another.

The diffusion area of the core, L^2, will be that of the moderator multiplied by the factor (1-f) or

10,100(1-0.976) = 242 cm^2. The effective multiplication factor may now be calculated:

$$k_{eff} = k_\infty \frac{e^{-B_g^2 \tau}}{1+L^2B_g^2}$$

$$= 2.02 \frac{\exp(-8.5\text{x}10^{-4}\text{x}3.54\text{x}10^2)}{1+2.42\text{x}10^2\text{x}8.5\text{x}10^{-4}}$$

$$= 1.24 \qquad 77$$

Reactor control

There are two somewhat unusual characteristics of a continuous flow reactor that affect its control. These are the diminished importance of the delayed neutrons because of their redistribution throughout the reactor, and the fact that their effect is also reduced as a result of the high turnover rate of the fuel inventory in the core. This will cause a substantial fraction of all but the shortest lived precursors to the delayed neutron emitters to be carried outside the reactor core with the gas, prior to their decay. Thus, a considerable fraction of the delayed neutrons will be emitted outside the core and will not contribute anything to the mean neutron lifetime in the reactor. However, a major portion of those delayed neutron carriers that have a relatively long half life will be carried around the circuit and back into the core again, and thus contribute to the mean neutron lifetime. Thus, delayed neutrons will play a role, albeit a diminished one, in the contribution to the reactor period. The problem will not be treated quantitatively here, but it should be realized that control of this type of reactor may be more difficult than that of a conventional one as a result of the diminished role of the delayed neutrons.

Fig. 11 shows the relationship between k_{eff} and f for the reactor under consideration, based on equation 77 (where both k_{∞} and L^2 are functions of f, as previously stated).

For effective reactor shutdown it should be possible to reduce k_{eff} to at least 0.95. From Fig. 11 this is seen to correspond to a value of f of 0.933.

Preferably, the poison used to reduce the thermal utilization in the core should be a solid at the core temperature. A high macroscopic absorption cross section is desirable, if not necessary. Hafnium carbide is one compound which fulfills both of these criteria. What is more, it is miscible with solid carbon and can therefore be incorporated in graphite stringers, if need be, as hafnium carbide is itself probably a poor structural material. Hafnium carbide has a melting point of 4160°K, a microscopic absorption cross section (per molecule) of 105 b

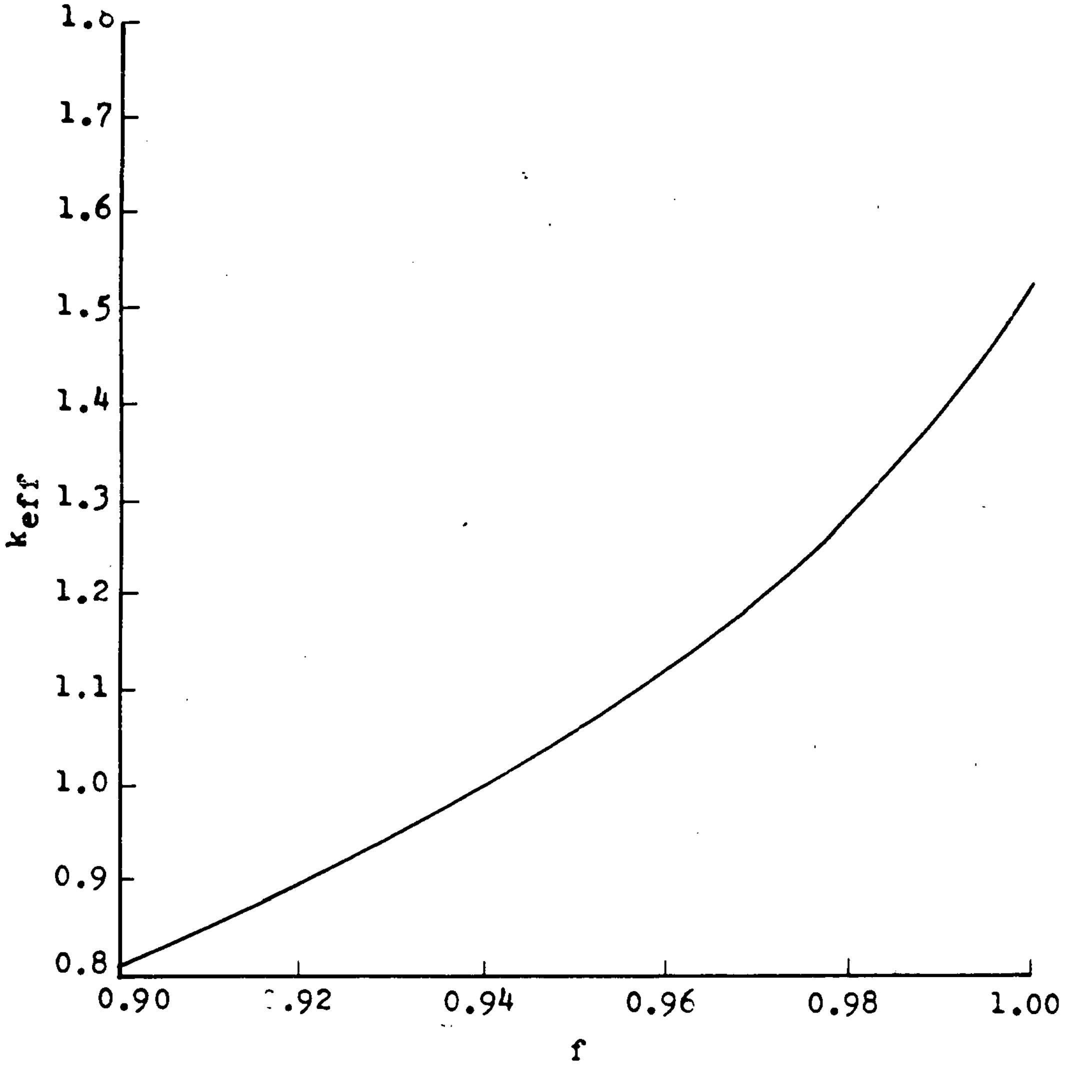


Fig. 11. Effective multiplication factor vs. thermal utilization

and a (computed) density of 12.5 g/cm^3.

The total volume of hafnium carbide needed in the core will be minute by comparison with the volume of moderator and fuel, hence the total core volume (or total moderator volume) will be considered unchanged. The poison will be assumed to be distributed throughout the moderator. Let the macroscopic absorption cross section of the mixture of moderator and poison be designated by Σ_{l+p}. For a thermal utilization of 0.933, 1/f will have a value of 1/0.933 = 1.07. Referring to equation 72:

$$\frac{1}{f} = \frac{V_{l+p}}{V_o} \frac{\Sigma_{l+p}}{\Sigma_o} 1.068 + 1.023 = 1.07$$

$$= 2 \frac{\Sigma_{l+p}}{5.85 \times 10^{-2}} 1.068 + 1.023 \qquad 78$$

Therefore,

$$\Sigma_{l+p} = 1.29 \times 10^{-3} \text{cm}^{-1} \qquad 79$$

As before, letting $\bar{N}$ designate the number of atoms of a species per unit volume of the mixture, further remembering the volume increase due to the poison is negligible:

$$\Sigma_{l+p} = \bar{N}_l \sigma_l + \bar{N}_p \sigma_p \approx \Sigma_l + \bar{N}_p \sigma_p$$

$$= 7.36 \times 10^{-5} + \bar{N}_p \times 105 \times 10^{-24} = 1.29 \times 10^{-3} \text{cm}^{-1} \qquad 80$$

Solving for $\bar{N}_p$ yields:

$$\bar{N}_p = \frac{1.29\times10^{-3}-7.36\times10^{-5}}{105\times10^{-24}} = 1.23\times10^{19} \text{ atoms/cm}^3 \quad 81$$

The total moderator volume will be twice the fuel volume of 0.964 m^3 or 1.925 $m^3 \Rightarrow 1.925\times10^6 cm^3$. The molecular weight of hafnium carbide is 190.61. The total weight of poison, w_p, in the core will therefore be:

$$w_p = \frac{\bar{N}_p V_1}{N_{Av}} M_p = \frac{1.23\times10^{19}\times1.925\times10^6}{6.03\times10^{23}} 190.61$$

$$= 7.5\times10^3 \text{ g} \quad 82$$

Ignoring the effect of thermal expansion on the density (of the order of a few percent), the volume of 7,500 grams of pure hafnium carbide will be 7,500/12.5 = 600 cm^3, which is quite minute by comparison with the moderator volume. This shows that the assumption made in considering the core volume unaffected by the poison is permissible.

In practice, the poison must be incorporated in control rods rather than being distributed throughout the core, but by placing these in such a way as to introduce the poison into the central region of the core, the effect of the poison will be enhanced, due to the greater statistical weight it will have in the region of higher flux. Thus, the preceding calculations based on uniform distribution of the poison will give a conservative result.

Because of the possible inaccuracy of the reactivity calculations and to incorporate a safety factor in the control system, a certain amount of redundancy is desirable. A safety factor of three or four would not be excessive. Accordingly, the total amount of hafnium carbide employed in the system (safety, shim and regulating rods) might easily be 4x7,500 = 30,000 g or 30 kg.

Shielding

Because of the high temperature of the core the radiological shielding of the reactor, which will be mostly concrete, must be thermally insulated from it. Therefore, the core must be clad with some sort of an insulator. Here it is proposed to use silicon carbide, because of its capability of withstanding extremely high temperatures.

As the reactor is pressurized it is also necessary to enclose the core in some sort of a pressure vessel, and because of the core temperature the pressure vessel must be outside the core insulating blanket.

In order that the average exit temperature of the gas from the core will be $3000^{o}K$, this must be the temperature of the gas leaving all channels, as this is the maximum temperature that the graphite is to be exposed to. Since the flux and hence the power generation is highest in the central regions of the core and lowest in the outermost channels, this must be compensated for by regulating the gas flow through the channels accordingly. By means of orifices at the inlet end of each channel the flow rate, and hence the temperature of the gas leaving that channel, can be regulated so as to give a uniform channel exit temperature. Therefore, the gas in the outermost unit cells will also be at 3000°K.

It is proposed to have the reflector clad with a 14 cm (minimum) thick layer of silicon carbide. In order to allow for thermal expansion of the core, a gap will be allowed for between the insulator and core. Five centimeters will suffice. When not taken up by the core expansion this gap can be filled by carbon dioxide or some other inert gas. The heat transmitted through the thermal insulator and pressure vessel must be removed. This can be done by cooling the pressure vessel by a stream of air or some other coolant fluid. Fig. 12 illustrates the thermal insulation arrangement.

If the cooling fluid is run concurrently to the fuel flow and allowed a maximum temperature of 300°C or 573°K, the temperature drop at the top of the core, from fuel to the outside of the pressure vessel, will be 3000-573

= 2427 degrees.

As may be seen from Fig. 13, the maximum diameter of core plus reflector is seven unit cell widths or 7x37.8 = 264 cm. Adding 14 cm of insulator onto this gives a diameter of 264+2x14 = 292 cm. Then, allowing for the 5 cm gap on either side, the inside diameter of the pressure vessel will be 292+10 = 302 cm. Allowing for a maximum stress, S_t, of 18,000 p.s.i. (assuming structural steel as construction material), then, if the pressure vessel is to contain a pressure of one hundred atmospheres or 1470 p.s.i., the wall thickness may be computed from the thin

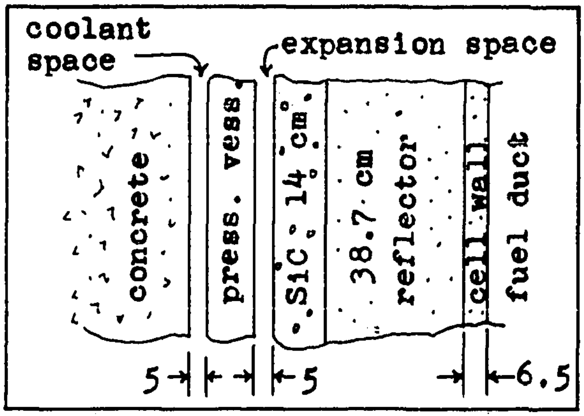
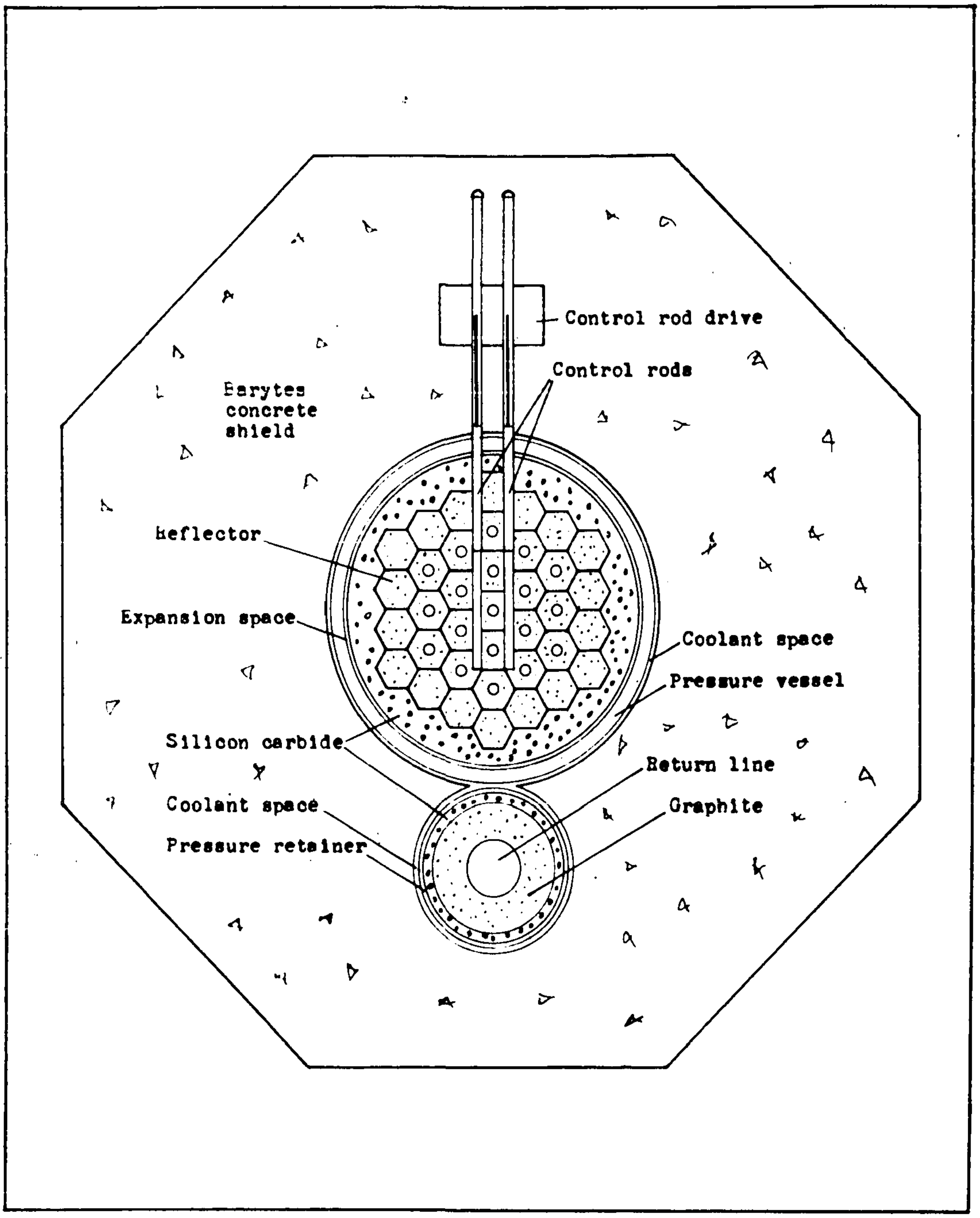


Fig. 12. Thermal insulation arrangement

Fig. 13. Cross section of core


Control rod drive
Control rods
Barytes concrete shield
Reflector
Expansion space
Coolant space
Pressure vessel
Silicon carbide
Return line
Coolant space
Graphite
Pressure retainer

wall formula:

$$t = \frac{pD}{2S_t} = \frac{1470x302}{2x18,000} = 12.3 \text{ cm} \qquad 83$$

This value corresponds to 4.85 inches and seems quite feasible, as large pressure vessels with walls up to eight inches in thickness have been constructed.

From the unit cell dimensions the distance from the outermost fuel duct to outer edge of reflector is seen to be 44.3 cm.

At the temperatures prevailing in each of the three materials, the graphite, silicon carbide and steel will have a thermal conductivity of 16, 8 and 26 pcu/hr ft deg K, respectively.

The corresponding thermal resistances of the materials will be in the ratio (44.3/16):(14/8):(12.3/26) or 2.77:1.75:0.462. When the total temperature differential of 2427 degrees is considered these ratios correspond to temperature drops of 1347, 850 and 242 degrees. The maximum temperature of the silicon carbide will be 3000-1347 = 1653^{o}K (Its melting point is 2873^{o}K). Accordingly, the maximum temperature of the pressure vessel will be 1653-850 = 803^{o}K or 530^{o}C.

The heat transfer losses from the core, q, based on a unit area of the pressure vessel will be (ignoring the curvature of the vessel surface):

$$q = \frac{k}{l}\Delta T = \frac{26}{12.3/30.5}\,242$$

$$= 15{,}600 \text{ pcu/hr ft}^2 \qquad 84$$

This corresponds to 4.5 kilowatts/ft^2. The pressure vessel surface (See Fig. 13) immediately surrounding the core is 3.02 x1.65 = 15.75 m^2 on the sides and, roughly, $(3.02/2)^2\pi = 7.19$ m^2 on the bottom, giving a total of 22.84 m^2 or 246 ft^2. Hence, the heat losses from the core, due to conduction, may be approximately estimated as 4.5x246 = 1,080 KW or 1.08 MW.

Next, the attenuation of radiation from the core in the reflector, thermal insulator and pressure vessel must be considered. Taking the core proper, its volume will consist of 1.925 m^3 of graphite and 0.964 m^3 of fuel, a total of 2.89 m^3. The mass of the graphite will be 1.925x1.52 = 2.92 metric tons (1.52 is the graphite density at the core temperature), and that of the fuel 110 kg, giving a total of 2.931 tons. The core density will thus be 2.931/2.89 = 1.015 g/cm^3. The volume source strength of the core, assuming a gamma energy release, E_γ, of

20 Mev/fission will be:

$$S_v = \frac{\frac{dN}{dt} E_\gamma}{V_c} = \frac{3.1x10^{19}x20}{2.89x10^6}$$

$$= 2.14x10^{14} \text{ Mev/cm}^3 \text{ sec} \qquad 85$$

The average energy of a gamma photon may be taken as 4 Mev. This corresponds to a mass absorption coefficient of 0.032 cm^2/g or a linear absorption coefficient of $0.032\rho = 0.032x1.015 = 0.0325 \text{ cm}^{-1}$. The relaxation length, λ, of the core will then be $0.0325^{-1} = 30.8$ cm. The source strength at the core surface will consequently be:

$$S_a = S_v \lambda = 2.14x10^{14}x30.8$$

$$= 6.6x10^{15} \text{ Mev/cm}^2 \text{ sec} \qquad 86$$

The reflector will have a relaxation length of 20.5 cm (corresponding to a density of 1.52 g/cm^3). Then the source strength at the surface of the reflector will be:

$$S_a' = S_a \frac{t}{\lambda} e^{-t/\lambda} = 6.6x10^{15}x\frac{37}{20.5} e^{-37/20.5}$$

$$= 9.8x10^{14} \text{ Mev/cm}^2 \text{ sec} \qquad 87$$

The silicon carbide insulating blanket has a density of 3.21 g/cm^3 and hence a relaxation length of

1/0.032x3.21 = 9.74 cm. Therefore, at its outer surface:

$$S_a'' = 9.8\text{x}10^{14}\,\frac{14}{9.74}\,e^{-14/9.74}$$

$$= 3.34\text{x}10^{14}\ \text{Mev/cm}^2\ \text{sec} \qquad 88$$

For the pressure vessel the relaxation length is 3.7 cm. Hence:

$$S_a''' = 3.34\text{x}10^{14}\,\frac{12.3}{3.7}\,e^{-12.3/3.7}$$

$$= 4\text{x}10^{13}\ \text{Mev/cm}^2\ \text{sec} \qquad 89$$

This is the surface source strength upon which the calculations of the biological shield thickness must be based. The shield material is assumed to be barytes concrete with a relaxation length of 10 cm. As before, designating the shield thickness by t and setting the maximum permissible isotropic surface source strength at the outside of the biological shield as 2000 Mev/cm^2 sec:

$$2000 = 4\text{x}10^{13}\,\frac{t}{10}\,e^{-t/10} \qquad 90$$

This equation has a solution at t = 272 cm, and this will be the minimum thickness of the biological shield. The shield thickness is indicated in Fig. 13 which is a horizontal cross section through the reactor.

The shield thickness was computed on the basis of attenuating the primary gamma flux from the core to a safe value. When concrete is used as a shielding material this is also sufficient for the necessary reduction of the secondary gamma and neutron fluxes.

C. Generating Section

Expansion nozzle

Sutton (32) gives a relationship for an ideal nozzle, linking the weight rate of flow through the nozzle to the nozzle throat area, the conditions prevailing in the gas, and the value of various gas parameters. In the MKS system this equation is:

$$q = A_t p_o \gamma \sqrt{\frac{\left[2/(\gamma+1)\right]^{(\gamma+1)/(\gamma-1)}}{\gamma R T_o/M}} \qquad 91$$

where,

q is the flow rate, kg/sec

A_t is the throat area, m^2

p_o is the chamber (i.e., core) pressure, newtons/m^2

T_o is the chamber temperature, ^oK

R is the universal gas constant, $8.31 x 10^3$ joules/kg mole deg

The symbols M and γ designate the molecular weight and specific heat ratio, as before.

Now, the flow rate, as computed on p. 45, was 13.6 kg mole/sec or, since M = 311 kg/kg mole, 13.6x311 = 4230 kg/sec. γ was 1.08 and the (p, T) conditions in the core are $1.013\text{x}10^7$ newtons/m^2 (equivalent to 100 atm) and 3000°K. The coefficient to A_t in equation 91 will then be:

$$1.013\text{x}10^7\text{x}1.08\sqrt{\frac{\left[2/(1.08+1)\right]^{2.08/0.08}}{1.08\text{x}8.31\text{x}10^3\text{x}3\text{x}10^3/3.11\text{x}10^2}}$$

$$= 2.19\text{x}10^4 \text{ kg/sec m}^2 \qquad \text{91a}$$

For a large nozzle, the energy conversion efficiency is about 0.96. Therefore,

$$A_t = \frac{1}{0.96}\,\frac{q}{2.19\text{x}10^4} = \frac{4.23\text{x}10^3}{2.19\text{x}10^4\text{x}0.96}$$

$$= 0.202 \text{ m}^2 \qquad 92$$

This corresponds to a throat diameter of 50.6 cm. It is then possible to calculate the nozzle exit diameter from the expansion ratio. Let the subscript x refer to the exit conditions.

Then,

$$\frac{A_t}{A_x} = \left(\frac{\gamma+1}{2}\right)^{(1/(\gamma-1))} \left(\frac{p_x}{p_o}\right)^{1/\gamma} \sqrt{\frac{\gamma+1}{\gamma-1}\left\{1-\left(\frac{p_x}{p_o}\right)^{(\gamma-1)/\gamma}\right\}}$$

$$= \left(\frac{2.08}{2}\right)^{1/0.08} \left(\frac{3}{100}\right)^{1/1.08} \sqrt{\frac{2.08}{0.08}\left\{1-\left(\frac{3}{100}\right)^{\frac{0.08}{1.08}}\right\}}$$

$$= 0.155 \qquad 93$$

Therefore, the exit diameter will be $50.6/0.155^{\frac{1}{2}} = 128.5$ cm. The half angle of divergence of the nozzle will be selected as 15°. Consequently, the length of the divergent portion of the nozzle will be $(128.5-50.6)/2 \tan 15^{\circ} = 145.2$ cm. The half angle of convergence is taken to be 45°. The corresponding length of the convergent portion would be 68.8 cm, if the transition from convergence to divergence were abrupt. In reality, the transition will be smooth and the total nozzle length a little longer than indicated by the above calculations. The actual dimensions are given in Fig. 15.

Finally, the ratio of the gas velocity at the nozzle exit, v_x, to that at the throat, v_t, may be computed from the relation:

$$\frac{v_x}{v_t} = \sqrt{\frac{\gamma+1}{\gamma-1}\left\{1-\left(\frac{p_x}{p_o}\right)^{(\gamma-1)/\gamma}\right\}}$$

$$= \sqrt{\frac{2.08}{0.08}\left\{1-\left(\frac{3}{100}\right)^{0.08/1.08}\right\}} = 2.44 \qquad 94$$

The gas velocity at the throat may also be computed:

$$v_t = \sqrt{\frac{2\gamma}{\gamma+1}\,\frac{RT_o}{M}} = \sqrt{\frac{2x1.08}{2.08}\,\frac{8.31x10^3x3x10^3}{3.11x10^2}}$$

$$= 289 \text{ m/sec} \qquad 95$$

Therefore,

$$v_x = 2.44v_t = 2.44x289 = 705 \text{ m/sec} \qquad 96$$

The nozzle outlet temperature will be:

$$T_x = T_o\left(\frac{p_x}{p_o}\right)^{(\gamma-1)/\gamma} = 3000\left(\frac{3}{100}\right)^{0.08/1.08}$$

$$= 2318^oK \qquad 97$$

At this temperature the velocity of sound in the gas, a, is:

$$a = \sqrt{\frac{\gamma RT}{M}} = (1.08\text{x}8.31\text{x}10^3\text{x}2.318\text{x}10^3/3.11\text{x}10^2)^{\frac{1}{2}}$$

$$= 259 \text{ m/sec} \qquad 98$$

The Mach number of the gas at the nozzle exit will then be $v_x/a = 705/259 = 2.72$.

Gas conductivity

The free electrons in the gas are considered to have a maxwellian energy distribution about their most probable energy E_o, corresponding to a scattering cross section σ_o. If the energy is written in terms of the most probable energy, and the basis of the distribution function is taken to be the flux density per unit energy interval (designated by $\phi(E/E_o)$), then the distribution function takes on the form:

$$\frac{\phi(E/E_o)}{\phi} = \frac{E}{E_o} e^{-E/E_o} \qquad 99$$

where ϕ is the total electron flux.

For a conservative result in computing the gas conductivity equation 48 may be used. The cross section is seen to have a 1/E variation (k^2 is proportional to E). Therefore, it follows that:

$$\sigma(E/E_o) = \sigma_o \frac{E_o}{E} \qquad 100$$

The effective maximum cross section may now be computed. It is easily seen that:

$$\bar{\sigma} = \frac{\int_0^\infty \phi(E/E_o)\sigma(E/E_o)d(E/E_o)}{\int_0^\infty \phi(E/E_o)d(E/E_o)} \qquad 101$$

Substituting from equations 99 and 100, and also replacing the integral in the denominator by its equivalent, the total flux ϕ, then:

$$\bar{\sigma} = \sigma_o \int_0^\infty e^{-E/E_o} d(E/E_o) = \sigma_o \qquad 102$$

The most probable electron energy at the nozzle exit conditions is that equivalent to the exit temperature of 2318°K or 3.22×10^{-13} erg. The square of the wave number corresponding to this energy will be:

$$k^2 = 2mE/\hbar^2 = 2\times9.1\times10^{-28}\times3.22\times10^{-13}/1.10\times10^{-54}$$

$$= 5.33\times10^{14} \text{ g/erg sec}^2 \qquad 103$$

From equation 48 the maximum effective cross section will be:

$$\sigma_o = \frac{4\pi}{k^2} = 4\pi/8.33\text{x}10^{13} = 2.36\text{x}10^{-14}\ \text{cm}^2 \qquad 104$$

Now, consider the dissociation of the cesium fluoride added to the gas. At the nozzle exit temperature and pressure the free fluorine will be monatomic rather than molecular. Therefore, the dissociation will be given by:

$$CsF(g) \rightleftarrows Cs(g) + F(g)$$

Fig. 9 gives the free energy function for the dissociation:

$$CsF(s) \rightleftarrows Cs(l) + \tfrac{1}{2}F_2(g)$$

The data from Fig. 9 must be combined with the free energies of vapourization of the substances not in the gas phase as well as the free energy of dissociation of fluorine, in order to get the appropriate free energy change for the dissociation of cesium fluoride in the gaseous phase.

The thermodynamic data used in the following computations was obtained from Quill (29) and Kelley (33).

As a first step consider the following series of equilibria at the normal boiling point of cesium under a pressure of one atmosphere:

$$\begin{array}{ll} CsF(l) \rightleftarrows Cs(l) + \tfrac{1}{2}F_2(g) & \Delta F = \Delta F_1 \\ CsF(g) \rightleftarrows CsF(l) & \Delta F = \Delta F_2 \\ Cs(l) \rightleftarrows Cs(g) & \Delta F = 0 \\ \hline CsF(g) \rightleftarrows Cs(g) + \tfrac{1}{2}F_2(g) & \Delta F = \Delta F_1 + \Delta F_2 \end{array}$$

From Fig. 9:

$$\Delta F_1 = \frac{\Delta F - \Delta H_{298}}{T} T + \Delta H_{298}$$

$$= 23x943-131,700 = 110,050 \text{ cal/g mole} \qquad 105$$

At 948°K the vapour pressure of CsF is $7x10^{-4}$ atm. Therefore:

$$\Delta F_2 = RT\ln P = 1.99x943\ln(7x10^{-4})$$

$$= -13,630 \text{ cal/g mole} \qquad 106$$

Hence,

$$\Delta F = \Delta F_1 + \Delta F_2 = 110,050 - 13,630$$

$$= 96,420 \text{ cal/g mole} \qquad 107$$

On the basis of data from Kelley (33) the heat of dissociation of cesium fluoride into gaseous cesium and diatomic fluorine was found to be given by the expression:

$$\Delta H_T = 129{,}306+0.16T+0.15\text{x}10^{-3}T^2+\frac{0.19\text{x}10^5}{T} \qquad 108$$

When this expression is substituted into the familiar relation:

$$\frac{d(\frac{\Delta F}{T})}{d(\frac{1}{T})} = \Delta H \qquad 109$$

then, subsequent integration yields:

$$\Delta F_T = 129{,}300-0.16T\ln T-1.5\text{x}10^{-4}T^2+9.5\text{x}10^3/T+IT \qquad 110$$

where I is a constant of integration. From the known value of the free energy of dissociation at 943°K it was evaluated at -33.7 cal/deg. The free energy of dissociation of gaseous cesium fluoride at the nozzle exit temperature, 2318°K, will be:

$$\begin{aligned}\Delta F_{2318} = 129{,}300 &- 0.16\text{x}2318\ln 2318\\ &- 1.5\text{x}10^{-4}\text{x}2318^2 + 9.5\text{x}10^3/2318\\ &- 33.7\text{x}2318 = 48{,}400 \text{ cal/g mole} \qquad 111\end{aligned}$$

The above value corresponds to dissociation into cesium gas and diatomic fluorine. The diatomic fluorine will further dissociate into monatomic fluorine. A corresponding computation for the fluorine, based on the heat and free energies of dissociation at 298°K being 18.3 and 14.2 Kcal/g atom, respectively, yields the result:

$$\Delta F_T = 18{,}267 - 0.98T\ln T + 0.17\times10^{-3}T^2 + 0.315\times10^5/T - 8.57T \qquad 112$$

At 2318°K the computed free energy is found to be:

$$\begin{aligned}\Delta F_{2318} = {} & 18{,}267 - 0.98\times2318 \ln 2318 \\ & + 0.17\times10^{-3}\times2318^2 + 0.315\times10^5/2318 \\ & - 8.57\times2318 = -18{,}320 \text{ cal/g atom}\end{aligned} \qquad 113$$

This means that the fluorine will be almost completely dissociated at that temperature.

Next, consider the following equilibria:

$$\begin{array}{ll} CsF(g) \rightleftarrows Cs(g) + \tfrac{1}{2}F_2(g) & \Delta F = 48{,}400 \text{ cal/g mole} \\ \tfrac{1}{2}F_2(g) \rightleftarrows F(g) & \Delta F = -18{,}320 \text{ cal/g mole} \\ \hline CsF(g) \rightleftarrows Cs(g) + F(g) & \Delta F = 30{,}080 \text{ cal/g mole} \end{array}$$

This represents the standard free energy of dissociation of the cesium fluoride at 2318°K. The equilibrium constant for this last reaction may now be computed.

$$K_1 = \frac{p_{Cs}p_F}{p_{CsF}} = e^{-\Delta F/RT} = \exp(-30{,}080/1.99\times2318) = 1.5\times10^{-3} \text{ atm.} \qquad 114$$

Let K_2 similarly designate the equilibrium constant for the dissociation:

$$Cs \rightleftarrows Cs^+ + e^-$$

Thus,

$$K_2 = \frac{p_+p_-}{p_{Cs}} \qquad 115$$

Cesium has an ionization potential of 3.87 v which is equivalent to 89,200 cal/g atom. The statistical weight factor g_eg_i/g_a will be $2x1/2 = 1$. Hence, from Saha's equation (equation 2, p. 8):

$$\log K_2 = -\frac{89,200}{4.573x2318} + \frac{5}{2}\log 2318 - 6.49$$

$$= -6.34; \quad K_2 = 4.58x10^{-7} \text{ atm.} \qquad 116$$

In computing the conductivity the following approximations will be made: a) that the resistance of the gas is due entirely to scattering by uranium tetrafluoride molecules. This will be very nearly true for a small content of cesium fluoride in the gas. b) That the partial pressure of fluorine will, for all intents and purposes equal that of nonionized cesium. This should be permissible because only a very small fraction of the free cesium atoms will be ionized. In other words, this is equivalent to assuming that the equilibrium of the cesium fluoride dissociation will be virtually unaffected by the formation of ions by a few of the cesium atoms. c) The final approximation that will be made is that the total pressure in the system will more or less equal the partial pressure of the uranium tetrafluoride. Again, this should be permissible for a small cesium fluoride content.

Now, from equation 114:

$$p_{Cs} = p_F = (K_1 p_{CsF})^{\frac{1}{2}} \qquad 117$$

Together with equation 115, this leads to:

$$p_+ = p_- = (K_2 p_{Cs})^{\frac{1}{2}} = K_2^{\frac{1}{2}} K_1^{\frac{1}{4}} p_{CsF}^{\frac{1}{4}} \qquad 118$$

From the ideal gas law $n_-/n = p_+/\pi$, where π is the total pressure in the system and, it will be remembered, n designates the molecular density of uranium tetrafluoride. Substitution of n_-/n into equation 20, p. 16, in terms of p_+/π where p_+ is written in the form given by equation 118, will give for the gas conductivity:

$$\eta_0 = \frac{e^2}{m v_- \sigma_0} \, \frac{K_2^{\frac{1}{2}} K_1^{\frac{1}{4}} p_{CsF}^{\frac{1}{4}}}{\pi} \qquad 119$$

The effective electron velocity, v_-, will be that corresponding to a most probable electron energy of 3.22×10^{-13} erg as previously computed. This will be equal to

$1.128x(2E/m)^{\frac{1}{2}} = 1.128(2x3.22x10^{-13}/9.1x10^{-28})^{\frac{1}{2}}$ $= 3x10^{7}$ cm/sec. 1.128 is the maxwellian conversion factor for the most probable to effective velocity conversion. Then, substituting the appropriate values in the MKS system into equation 119 (with the exception of the equilibrium constants and pressures, which are in units of atmospheres):

$$\eta_o = \frac{1.6^2x10^{-38}}{9.1x10^{-31}x3x10^{5}x2.36x10^{-18}}$$

$$x\frac{(4.58x10^{-7})^{\frac{1}{2}}(1.5x10^{-3})^{\frac{1}{4}}}{3}\,p_{CsF}^{\frac{1}{4}} = 1.77p_{CsF}^{\frac{1}{4}} \text{ mho/m} \qquad 120$$

If it is assumed that the approximations made in deriving the conductivity equation hold up reasonably well up to a cesium fluoride content of 5 percent (this should be within reason, as a conservative value of the cross section was used), then this will correspond to a cesium fluoride partial pressure of 0.15 atm. The conductivity will then be $1.77x0.15^{\frac{1}{4}} = 1.1$ mho/m. This is in the absence of a magnetic field.

Generator

In the generator passage the kinetic energy of the jet is partly converted into electrical energy by virtue of the magnetic field there. If the increase of the potential energy of the gas, due to its elevation, is neglected, the first law of thermodynamics as applicable to continuous flow states that:

$$\frac{\Delta V^2}{2} = - W_n - \Delta h \quad \text{(Note the analogy to a turbine)} \qquad 121$$

where $- W_n$ depicts the net electrical work done by the gas on its surroundings. The energy of the current induced in the gas jet will be partly dissipated within the gas itself and partly without. The resulting ohmic losses within the gas will raise its temperature and hence its enthalpy ($\Delta h = C_v \Delta T + R \Delta T$ on a per mole basis). If the gas were a perfect conductor there would be no ohmic losses within it and no resultant temperature rise. Consequently, its enthalpy increase would be zero and the total electrical work done would be that on the surroundings and $-W_n = \Delta V^2/2$.

It is thus seen that $\Delta V^2/2$ corresponds to the total electrical work done and, with a real gas, part of this work is dissipated within the gas as ohmic losses and manifests itself as increased enthalpy of the exit gas. (The generator is seen to partly negate the effect of the nozzle of converting the enthalpy of the gas into kinetic energy of the jet.)

Referring to Fig. 14a: Consider a column of gas, a unit area in cross section and of length L (the electrode separation). Let the electrode potential be E_0. The partly charged electrodes will oppose the inductive force on the charged particles so the net equivalent electric field will

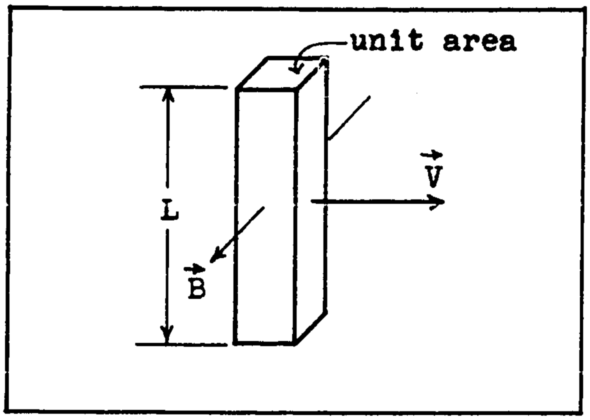


Fig. 14a. Explanatory diagram for generator principles

be $(BLV - E_o)/L$. Then, from equation 44, p. 24:

$$j = \frac{\eta_o}{1 + \omega^2/\nu^2} \frac{BLV-E_o}{L} \qquad 122$$

The total potential drop around the circuit will be $(BLV - E_o) + E_o = BLV$. The resulting total power dissipation, P_t ($= \rho Ld(-W_n - \Delta h)/dt$), for the power extracted from the kinetic energy of the gas column will be:

$$P_t = \frac{\eta_o}{1 + \omega^2/\nu^2} \frac{BLV-E_o}{L} BLV \qquad 123$$

Furthermore, from equation 121,

$$P_t = -\tfrac{1}{2}\rho L\, dV^2/dt = -L\rho V\, dV/dt \qquad 124$$

Therefore,

$$\frac{\eta_o BV}{1 + \omega^2/\nu^2}(BLV - E_o) = -L\rho V \frac{dV}{dt} \qquad 125$$

Because the channel is of uniform cross section it follows from the continuity equation that ρV = const. Let $\rho VL = k_1$. Replacing ν by $n\sigma v$ and substituting for n by its equivalent k_2/V (nV is constant = k_2, because n is proportional to ρ), then the expression takes on the form:

$$\frac{\eta_o(B^2LV^2 - BE_oV)}{1 + \omega^2V^2/k_2^2\sigma^2v^2} = -k_1\frac{dV}{dt} \qquad 126$$

Hence,

$$-\frac{\eta_o}{k_1}dt = \frac{1 + (\omega^2/k_2^2\sigma^2v^2)V^2}{B^2LV^2 - BE_oV}\,dV \qquad 127$$

Let,

$$\omega^2/k_2^2\sigma^2v^2 = a$$

$$B^2L = b \qquad 128$$

$$BE_o = c$$

Therefore,

$$-\frac{\eta_o}{k_1} t = \int \frac{1 + aV^2}{bV^2 - cV}\, dV = \left(\frac{1}{c} + \frac{ac}{b^2}\right)\ln(bV-c)$$

$$-\frac{1}{c}\ln V + \frac{a}{b^2}(bV-c) + C \qquad 129$$

where C is a constant of integration whose value is dependent on the conditions of the gas entering the generator channel.

Inspection of equation 128 will show that under most conditions the constant "a" will be very small. The dominant terms in equation 129 will be $\frac{1}{c}\ln(bV-c)$ and $-\frac{1}{c}\ln V$. Then, equation 129 reduces to:

$$-t \approx \frac{k_1}{\eta_o}\frac{1}{c}\left(\ln\frac{bV-c}{V} + C'\right) \qquad 130$$

where C' is the modified constant corresponding to the boundary conditions that t = 0 when V equals the velocity of the gas entering the generator.

It is proposed to establish the magnetic field in the generator section by means of superconductive coils constructed of a niobium-tin alloy. The alloy becomes superconductive at 18°K and has a threshold field value at 0°K of 10 w/m^2.

By maintaining the coils at or slightly below the boiling point of liquid helium at 4.2°K, the threshold field will be:

$$B_T = B_0(1-(\frac{T}{T_t})^2) = 10(1-(\frac{4.2}{18})^2)$$

$$= 9.45 \text{ w/m}^2 \tag{131}$$

where T_t is the transition temperature in the absence of a magnetic field and B_0 the value of the threshold field at $0^\circ K$.

If the field is to be established by sending a current pulse through the coils, and the maximum feasible magnitude of such a pulse is taken to be $3x10^6$ amperes (pulses of this order of magnitude are used in fusion research), then the minimum radius, (r), of the conductor must be:

$$r = \frac{\mu i}{2\pi B_T} \quad \frac{12.57x10^{-7}x3x10^6}{2\pi x9.45} \approx 0.0632 \text{ m} \tag{132}$$

where the permeability, μ, is taken to be that of empty space and i represents the current.

Now, for a coil of an infinite number of turns where n represents the number of turns per meter, the flux density, B, is given by the equation:

$$B = \mu n i \tag{133}$$

For a short coil consisting of a finite number of turns, where the length of the coil in the direction of the field is comparable to its diameter the flux density is about one-half of that of an infinite coil.

Therefore, if B is to be $9.45\ w/m^2$:

$$n = \frac{2B}{\mu i} = \frac{2x9.45}{12.57x10^{-7}x3x10^{6}} = 5 \text{ turns/m} \qquad 134$$

The generator passage will be of a square cross section, equal in area to the cross sectional area of the exhaust end of the nozzle which is 1.30 m^2. A cross section of the generator passage will thus be $1.30^{\frac{1}{2}} = 1.14$ m on the side. It must be thermally insulated in a similar fashion to the reactor core (and nozzle). A 14 cm thick silicon carbide insulation, together with about 6 cm of graphite would be needed. Allowing another 10 cm for cladding (this would have to be nonmagnetic) and air gap and a further 15 cm on either side for the liquid helium tank and insulation, and finally 6 cm for the conductor radius, the final diameter (or width) of the coil will be 1.14 +2(14+6+10+15) = 2.04 m. The coil would have to be at least this long and preferably a little longer, thus consisting of at least $5x2.04 \approx 10$ turns. At the ends of the generator passage the conductors would have to diverge to either side to make room for the nozzle and heat exchanger.

The value of the constants in equation 129 may now be computed. The (p,V) conditions prevailing at the nozzle exit are ($3.09\text{x}10^5$ n/m^2, $2.318\text{x}10^3$ ^{o}K). The molecular density per m^3 will be:

$$n = \frac{pN_{Av}}{RT} = \frac{3.09\text{x}10^5\text{x}6.03\text{x}10^{26}}{8.31\text{x}10^3\text{x}2.318\text{x}10^3}$$

$$= 9.5\text{x}10^{24} \text{ molecules/m}^3 \qquad 135$$

At the nozzle exit:

$$k_2 = nV = 9.5\text{x}10^{24}\text{x}7.05\text{x}10^2 = 6.7\text{x}10^{27} \text{ molecules/m}^2\text{sec} \qquad 136$$

As before, let M signify molecular weight:

$$k_1 = \rho VL = \frac{n}{N_{Av}} MVL = \frac{k_2}{N_{Av}} ML$$

$$= \frac{6.7\text{x}10^{27}}{6.03\text{x}10^{26}} 3.11\text{x}10^2\text{x}1.14 = 3.94\text{x}10^3 \text{ moles/sec m} \qquad 137$$

From equation 9:

$$\omega = \frac{Be}{m} = \frac{9.45\text{x}1.6\text{x}10^{-19}}{9.1\text{x}10^{-31}} = 1.66\text{x}10^{12} \text{ sec}^{-1} \qquad 138$$

Then,

$$a = \omega^2/k_2^2\sigma^2v^2$$

$$= (1.66\text{x}10^{12}/6.7\text{x}10^{27}\text{x}2.36\text{x}10^{-18}\text{x}3\text{x}10^5)^2$$

$$= 1.022\text{x}10^{-7} \qquad 139$$

The value of "a" is seen to be small enough so that it can be omitted.

$$b = B^2L = 9.45^2 \times 1.14 = 102 \qquad 140$$

The selection of a value for the constant c is somewhat more of a problem. The potential across the electrodes can never be greater than BLV_e where V_e is the exit velocity of the gas from the generator. If all the kinetic energy of the jet were to be converted into electrical energy the exit velocity would have to be zero, but this implies a zero electrode potential and no electric power dissipation outside the generator, i.e., all the kinetic energy would be transformed back into enthalpy of the gas.

On the other hand, if the exit velocity is high, only part of the kinetic energy of the jet is converted to electric energy, with the limiting case of no conversion when the exit and entry velocities are the same. It can easily be shown that the electric power dissipated externally to the generator is at a maximum when the exit velocity is one-half the entrance velocity, corresponding to a 75 percent utilization of the kinetic energy of the gas jet. Under these conditions, when the electrode potential corresponds to the exit gas velocity, the electric energy dissipated in the external circuit is only one-half of the total kinetic energy of the jet. This represents the limiting case and, as can be seen from equation 130, corresponds to an infinite deceleration time, that is, an infinite generator duct length. If the electrode potential is reduced to one-half of the mean potential induced in the gas, the net conversion of kinetic energy to electric power in the external circuit is reduced to 37.5 percent, but the generator length is cut down to a reasonable magnitude.

All the foregoing arguments presuppose a constant gas conductivity, and, in the case being considered, this is very nearly true. This is because, in this particular case, the effect of the increase in temperature of the gas (due to the ohmic losses within it) just about compensates for the rise in pressure, which affects the ion concentration of the gas adversely.

The problem, then, is to select the electrode potential and exit velocity in such a manner as to balance the conflicting extremes of no energy conversion vs. an infinitely long generator.

Because of the conservative value used here for the gas conductivity, it was decided to let the exit velocity be as high as 400 m/sec (The limiting case is 352.5 m/sec.) and the electrode potential was selected as 2710 volts (The limiting value for a 400 m/sec exit velocity is 3950 volts.).

The value of c will be:

$$c = BE_0 = 9.45x2710 = 25,600 \qquad 141$$

Referring to equation 130 it is seen, that in order to fulfill the boundary conditions, that at $t = 0$ $V = 705$ m/sec, $C' = 2.303 \log ((102 \times 705 - 25{,}600)/705) = 1.81749$. The equation then becomes:

$$t = \frac{3940 \times 2.303}{1.1 \times 25{,}600} \left(1.81749 - \log \frac{102V - 25{,}600}{V}\right)$$

$$= 0.322\left(1.81749 - \log \frac{102V - 25{,}600}{V}\right) \qquad 142$$

The above relationship is plotted in Fig. 14b. The time required to reduce the gas velocity to 400 m/sec is 76.5 milliseconds. Graphical integration of the curve and subsequent calculation of the mean gas velocity showed it to be 502 m/sec. Therefore, the length of the duct will be $5.02 \times 10^2 \times 76.5 \times 10^{-3} = 38.4$ m.

Because the gas leaves the nozzle at almost exactly the same temperature (2318°K) as it entered the reactor (2320°K), it follows from the first law of thermodynamics that the kinetic energy of the gas mass leaving the nozzle each second (13.6 kg moles) is the same as the amount of heat released in it in the reactor, that is, the kinetic energy of the gas mass entering the generator per second is 10^9 joules.

The rate of conversion of kinetic to electric energy in the generator is:

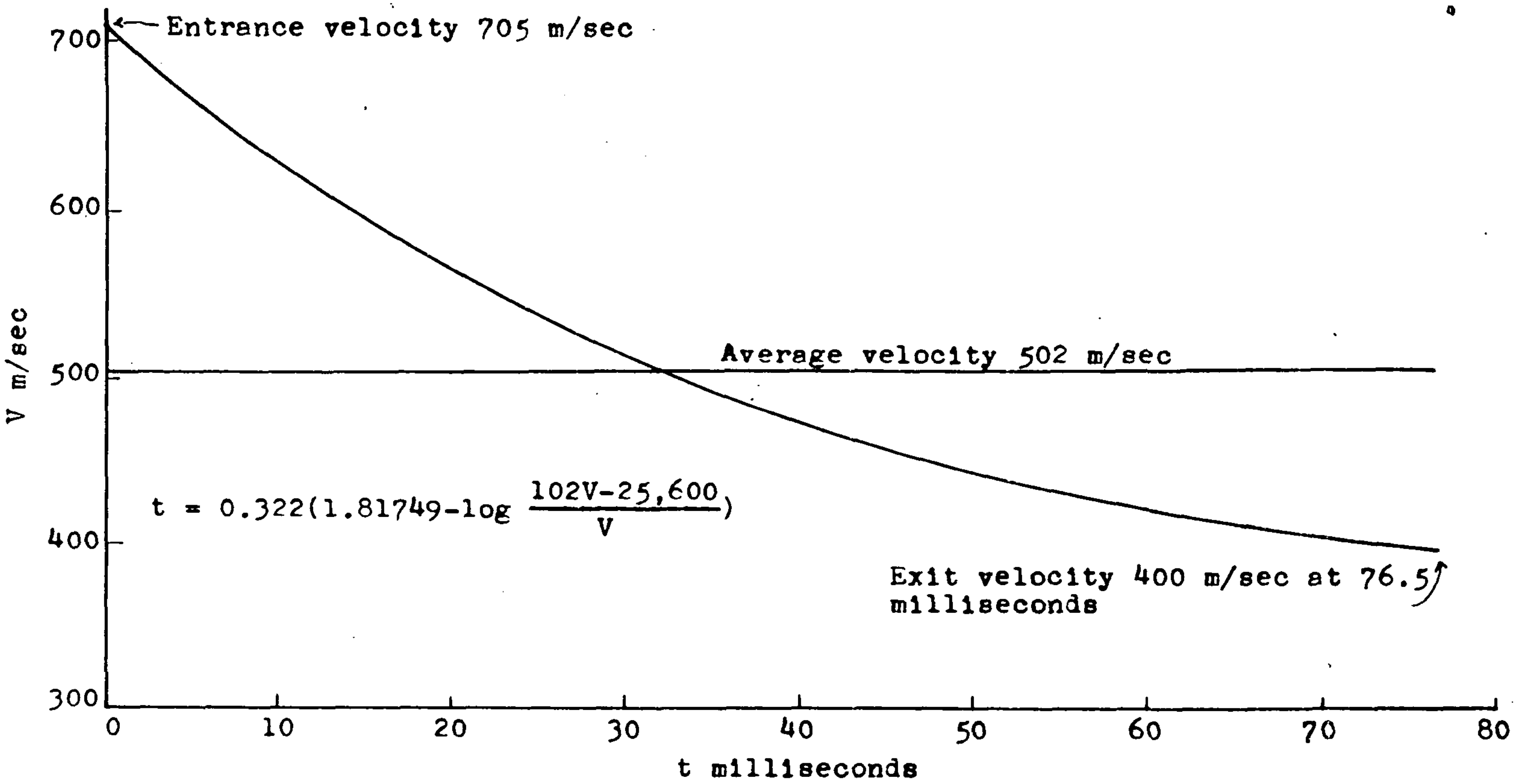


Fig. 14b. Gas velocity as a function of time spent in generator

$$P = \tfrac{1}{2}qM(V_1^2 - V_e^2) = \frac{13.6\text{x}311}{2}(705^2 - 400^2)$$

$$= 6.8\text{x}10^8 \text{ watts} \qquad 144$$

The mean electromotive force induced in the gas jet, corresponding to its mean velocity, is 9.45x1.14x502 = 6075 v. As the electrode potential is 2710 v, the electric power in the external circuit will be:

$$P_e = \frac{E_o}{BL\bar{V}} P = \frac{2710}{6075} 6.8\text{x}10^8$$

$$= 3.04\text{x}10^8 \text{ watts} \qquad 145$$

The remainder of the electrical power will be dissipated as ohmic losses in the gas and will manifest itself as an enthalpy increase. The enthalpy increase will correspond to $(6.8-3.04)10^8 = 3.76\text{x}10^8$ watts. It will be recalled that the enthalpy increase in the reactor, of 10^9 watts, corresponded to a temperature rise of 680 degrees. Since ideal gas behavior is being assumed, the temperature rise is directly proportional to the increase in gas enthalpy. It follows that the temperature rise due to the ohmic losses will be $(3.76\text{x}10^8/10^9)680 = 255$ degrees. The exit temperature will thus be $2318+255 = 2573^\circ$K. The gas in the generator has a supersonic velocity and consequently, it need not be at a uniform pressure. As mentioned before, the generator acts partly as a contraction nozzle, wherein part of the kinetic energy is converted adiabatically into thermal energy. The exit pressure is computed in the conventional manner (equation 97).

$$p = 3\left(\frac{2573}{2318}\right)^{1.08/0.08} = 12.3 \text{ atm} \qquad 146$$

It should be stressed that the value used for the gas conductivity corresponds to the maximum possible theoretical value of the electron scatter cross section of the uranium tetrafluoride molecule. Should it prove to be less than this value the conductivity will be greater and the generator length proportionately shorter. Another factor which could also increase the conductivity would be the direct ionization of cesium fluoride molecules to form molecular ions and free electrons. This has been ignored and it is not known to what degree this might enhance the conductivity. Finally, some additional contribution to the conductivity might be expected as a result of ionization by radiation and fission fragments.

With regard to the effect of the electron scatter cross section, it may be mentioned that data available for other tetrahalide molecules suggests that it is probably only about one-fourth of the maximum theoretical value. This would shorten the generator duct length to 9.65 m, which is quite an improvement. The length of the generator is doubly important because it also has a bearing on the fuel inventory.

Because the generator duct is of a uniform cross sectional area, it follows from the continuity equation that the density of the gas must be inversely proportional to its velocity at any given point. Therefore, the mean density must correspond to the mean velocity.

From equation 135 the molar density at the generator entrance is seen to be $9.5\text{x}10^{24}/6.03\text{x}10^{26} = 0.01575$ kg moles/m^3. Then the mean density will be $0.01575(705/502) = 0.022$ kg moles/m^3 or $0.022\text{x}311 = 6.86$ kg/m^3. Since the cross sectional area of the conduit is 1.30 m^2 and its length 38.4 m, the fuel inventory in the generator will be $6.86\text{x}1.30\text{x}38.4 = 353$ kg. This seems quite an impressive quantity but it should be kept in mind that it corresponds to the theoretical minimum of the conductivity and the actual quantity will probably be around four times less than this.

Another factor which has also been neglected in the generator calculations is the effect of the pressure gradient on the current.

The generator duct and the fuel contained therein must, of course, form a subcritical system. Because there is no moderator in the duct the prevailing mode of fission would be fast fission.

The critical dimensions of such a system are of a similar magnitude as the mean free path of a neutron. The macroscopic absorption cross section for a molar density of 0.485 moles/m^3 was computed to be $6.04\text{x}10^{-2}$ cm^{-1}. (equation 62) Therefore, the thermal macroscopic cross section in the generator will be approximately $(0.0157/0.485)6.04\text{x}10^{-2} = 1.96\text{x}10^{-3}$ cm^{-1}. A further correction must be made for the change in cross section from the thermal value of 170 b to a fast fission value of, say, 10 b, giving a macroscopic cross section of $(10/170)1.96\text{x}10^{-3} = 1.15\text{x}10^{-4}$ cm^{-1}. This would correspond to a maximum linear dimension of approximately 10^4 cm or 100 m.

D. Heat Exchanger and Compressor

The kinetic energy still left in the gas leaving the generator each second is seen to be $10^9 - 6.8\text{x}10^8 = 3.2\text{x}10^8$ joules. At the inlet end of the heat exchanger the pipes through which the gas is to flow must be flared so as to present a minimum obstruction to the gas and reduce shock wave formation (See Fig. 15). The pipes will then taper to a lesser diameter and further slow down the gas jet.

The maximum contraction in the cross sectional flow area is realized when the gas is adiabatically slowed down to sonic velocity. Letting the subscript 1 refer to the conditions of the gas leaving the generator, then the following relationship can easily be derived from equations 98 and 121:

$$T = \frac{MV_1^2 + 2C_pT_1}{2C_p + \gamma R}$$

$$= \frac{3.11\text{x}10^2\text{x}16\text{x}10^4+2\text{x}1.08\text{x}10^5\text{x}2.573\text{x}10^3}{2\text{x}1.08\text{x}10^5+1.08\text{x}8.31\text{x}10^3}$$

$$= 2680^{\circ}\text{K} \tag{147}$$

where T is the temperature of the gas at the point where it is slowed down to sonic velocity. At this temperature the velocity of the gas will be (equation 98):

$$V = (1.08\text{x}8.31\text{x}10^3\text{x}2.68\text{x}10^3/3.11\text{x}10^2)^{\frac{1}{2}}$$

$$= 278\ \text{m/sec} \tag{148}$$

The corresponding pressure is found to equal $12.3(2680/2573)^{1.08/0.08} = 20.9$ atm ($\simeq 2.12\text{x}10^6\ \text{n/m}^2$). The density is computed in the usual way:

$$= \frac{2.12\text{x}10^6\text{x}3.11\text{x}10^2}{8.31\text{x}10^3\text{x}2.68\text{x}10^3} = 29.6\ \text{kg/m}^3 \tag{149}$$

The total cross sectional flow area at the entrance to the heat exchanger may now be computed from the continuity equation:

$$A = \frac{A_1 V_1 \rho_1}{V \rho} = \frac{1.30 x 400 x 18.1}{29.6 x 278}$$

$$= 1.14 \text{ m}^2 \qquad 150$$

If it is not feasible to run the gas through the exchanger at such a high velocity the gas would have to be slowed down further prior to entering the exchanger. This, however, would require an increase in the cross sectional flow area needed to pass the given quantity of gas per unit time.

Cooling the gas at constant pressure to 2075°K will permit the cross sectional flow area to be further reduced. If there were no friction losses in the gas the velocity would be unchanged and the kinetic energy of the gas would be $(278/705)^2 = 0.155$ times its original kinetic energy. Thus, since the stagnation temperature at a velocity of 705 m/sec was some 680 degrees higher than the prevailing gas temperature, then the stagnation temperature of the gas emerging from the heat exchanger would be 0.155x680 + 2075 = 2180°K.

In practice there will be considerable friction losses in the exchanger, the exact magnitude of which will depend on the hydraulic radius of the individual ducts. Since this will vary it is difficult to undertake an accurate calculation without complete design of the heat exchanger.

The exchanger can be so designed as to operate at a constant pressure if the friction losses are compensated for by the deceleration of the gas. Let the mean temperature in the exchanger be taken as the arithmetical average between exit and entry temperatures, that is, 2380°K. The mean density will then be 29.6(2680/2380) = 33.2 kg/m^3. The velocity will also be somewhat reduced to a mean velocity of approximately 260 m/sec. This indicates (from equation 150) that the mean cross sectional flow area will be (29.6/33.2)(278/260) = 0.955 times smaller than that where the gas has been reduced to sonic velocity. Thus it will be 0.955x1.14 or 1.09 m^2.

Now, let the mean diameter (i.d.) of each pipe be selected as 0.05 m (i.e., 2 inches). The inlet end of each pipe must be flared to a square cross section so they can be arranged in such a way as to present no flat surface normal to the jet of incoming gas. The ratio of the cross sectional area at the inlet to its mean value is (1.30/1.09) = 1.19. Hence, the inlet end of each pipe will have a cross sectional flow area of $1.19(\pi/4)0.05^2 = 23.4\text{x}10^{-4}\text{m}^2$. This corresponds to a square that is 4.74 cm on the side. As the generator is 1.14 m on the side there will be room for a bank of 24 by 24 pipes (24 = 114/4.74) or 576 pipes. The mean periphery (based on i.d.) of each pipe is

$0.05\pi = 0.157$ m and this represents the heat exchange surface available per unit length for each pipe. The surface for the whole bank of pipes will be 576 times this or 90.5 m^2 per unit length.

Next, the heat transfer coefficient of the gas must be determined. This will have to be done theoretically from basic principles as no experimental data is available.

The atomic radii of monovalent fluorine and tetravalent uranium are given as 1.33 and 0.97 Angstrom units, respectively. The molecular radius, a, of the uranium tetrafluoride molecule will therefore be 1.33 + 0.97 = 2.30 A. Hence, the collision cross section of the molecule may be computed:

$$\sigma = 4\pi a^2 = 4\pi \times 2.30^2 \times 10^{-20} = 6.6\times10^{-19}\ m^2 \qquad 151$$

It should be emphasized that this is not the same quantity as the electron scatter cross section that has been previously calculated. The molecular mass is M/N_{Av} $= 311/6.03\times10^{26} = 5.16\times10^{-25}$ kg. Now, the mean molecular velocity is given by kinetic theory as being:

$$\bar{v} = \left(\frac{8kT}{\pi m}\right)^{\frac{1}{2}} = \left(\frac{8\times1.38\times10^{-23}\times2.38\times10^{3}}{5.16\times10^{-25}\,\pi}\right)^{\frac{1}{2}}$$

$$= 4.035\times10^{2}\ m/sec \qquad 152$$

where k is the Boltzman constant.

With this data it is possible to find the viscosity at the given temperature, as predicted by kinetic theory:

$$\mu = \frac{1}{3}\frac{m\bar{v}}{\sigma} = \frac{5.16\text{x}10^{-25}\text{x}4.035\text{x}10^{2}}{3\text{x}6.6\text{x}10^{-19}}$$

$$= 1.052\text{x}10^{-4} \text{ kg m/sec} \qquad 153$$

The thermal conductivity is obtained from the relationship:

$$k = \mu \frac{C_v}{M} = 1.052\text{x}10^{-4}\ \frac{99.68\text{x}10^{3}}{311}$$

$$= 3.38\text{x}10^{-2} \text{ joules/m sec deg} \qquad 154$$

Then, tabulating the pertinent parameters:

$$\begin{aligned} D &= 0.05 \text{ m} \\ V &= 260 \text{ m/sec} \\ \rho &= 33.2 \text{ kg/m}^3 \\ \mu &= 1.052\text{x}10^{-4} \text{ kg m/sec} \\ k &= 3.38\text{x}10^{-2} \text{ joules/m sec deg} \\ c_p &= 3.47\text{x}10^{2} \text{ joules/deg kg} \end{aligned} \qquad 155$$

The Reynolds and Prandtl numbers are:

$$Re = \frac{DV\rho}{\mu} = \frac{5\text{x}10^{-2}\text{x}2.6\text{x}10^{2}\text{x}33.2}{1.052\text{x}10^{-4}}$$

$$= 4.1\text{x}10^{6} \qquad 156$$

$$Pr = \frac{c_p \mu}{k} = \frac{3.47\times10^{2}\times1.052\times10^{-4}}{3.38\times10^{-2}} = 1.08 \qquad 157$$

Substituting into the Dittus-Boelter equation gives the individual heat transfer coefficient:

$$h = 0.023 \frac{k}{D} Re^{0.8} Pr^{0.4}$$

$$= 0.023 \frac{3.38\times10^{-2}}{5\times10^{-2}} (4.1\times10^{6})^{0.8} (1.08)^{0.4}$$

$$= 7115 \text{ joules/m}^2 \text{ sec deg} \qquad 158$$

It is proposed to use a liquid metal as a coolant fluid. Since the heat transfer coefficient would be very much higher than that of the gas, virtually all of the temperature drop would take place across the boundary layer in the gas.

It follows that for a reasonably good first approximation the exchanger design may be based on the properties of the gas only.

Next, the mean temperature of the coolant fluid requires some consideration. The less it is, the smaller the heat exchanger, but the lower the quality of the heat rejected, and vice versa. A happy medium seems to be the halfway mark between the gas temperature and the ambient temperature of the surroundings which will serve as the ultimate heat sink. Say, then, that the temperature drop, ΔT, across the gas film is taken as one thousand degrees. The overall temperature drop will be somewhat greater than this but not much, for the reasons explained above. The rate of heat rejection is:

$$Q = -qC_p\,(T_2 - T_1) = 13.6x1.08x10^5(2680-2075)$$

$$= 8.8x10^8 \text{ watts} \qquad 159$$

Hence, the heat exchange area, A, must be:

$$A = \frac{Q}{h\,\Delta T} = \frac{8.8x10^8}{7.115x10^3x10^3} = 124 \text{ m}^2 \qquad 160$$

Then the length of the exchanger must be:

$$L = \frac{124}{90.5} = 1.37 \text{ m} \qquad 161$$

Friction losses must also be considered. The Fanning friction factor is obtained from a friction factor vs. Reynolds number plot and found to be $2.4x10^{-3}$ for a Reynolds number of $4.1x10^6$. The friction losses per kg, w, will then be given approximately by the Fanning equation:

$$w = 2f\,\frac{V^2L}{D} = 2x2.4x10^{-3}\,\frac{2.6^2x10^4x1.37}{0.05}$$

$$= 8.9x10^3 \text{ joules/kg} \qquad 162$$

$$W = qMw = 13.6 \times 311 \times 8.9 \times 10^3 = 3.76 \times 10^7 \text{ watts} \qquad 163$$

This amount of energy will show up as additional heat that must be removed, necessitating a lengthening of the exchanger over and above the previously computed value to a length of $((8.8 + 0.376)/8.8)1.37 = 1.43$ m. This in turn will increase the friction losses by $(1.43/1.37)3.76 \times 10^7 = 3.92 \times 10^7$ watts (or 9.26×10^3 joules/kg), again requiring a slight increase in length, and so on, approaching a finite limit in both cases. The limiting value is close enough to that computed by the second approximation above so that the latter value may be used.

The expression for the exit velocity is derived from an energy balance of the friction losses and kinetic energy of the gas:

$$V = (V_1^2 - 2w)^{\frac{1}{2}} = (2.78^2 \times 10^4 - 2 \times 9.26 \times 10^3)^{\frac{1}{2}}$$

$$= 242 \text{ m/sec} \qquad 164$$

The mean velocity will be given accurately enough by the arithmetical average of the entrance and exit velocities or $(278 + 242)/2 = 260$ m/sec, which is the value that was used in the heat transfer calculations.

Practically all the kinetic energy left in the gas will be available for its recompression. By the same argument as stated on p. 103, the adiabatic temperature rise of the gas will be:

$$T = (242/705)^2 680 = 80 \text{ deg K} \qquad 165$$

It does not matter whether the conversion of kinetic energy to gas enthalpy takes place before, during or after the additional compression of the gas by the compressor, as enthalpy is a state function and the net result will always be the same. For ease of calculation assume that the energy conversion in question all takes place before the gas enters the compressor. It would then be some eighty degrees hotter than when it leaves the exchanger, and thus be at a temperature of 2075 + 80 = 2155°K. The gas is to enter the reactor at 2320°K (corresponding to a pressure of 100 atmospheres) and therefore the work of the compressor, W_n, will be:

$$W_n = qC_p(T_2 - T_1) = 13.6 \text{x} 1.08 \text{x} 10^5(2320 - 2155)$$

$$= 2.44 \text{x} 10^8 \text{ watts} \qquad 166$$

The power required to drive the compressor will have to come from the heat rejected by the gas. This requires another heat exchanger (unless the liquid metal in the primary heat exchanger is boiled directly) and a turbine to drive the compressor. The overall efficiency of the secondary system would have to be 2.44/(8.8 + 0.376) = 0.266, which should be easily attainable considering the high temperature at which the primary heat (i.e., from the generator heat exchanger) is rejected. With an efficiency of only 26.6 percent it might even be possible to reject the heat from the secondary system at a high enough temperature (100-200°C) to give it further value as processing heat for a chemical plant.

The physical size of the compressor can now be roughly estimated. The velocity head of the gas is capable of increasing the gas temperature from 2075 to 2155°K, as computed previously, and this corresponds to a compression ratio of $(2155/2075)^{1.08/0.08} = 1.655$. Since the overall compression ratio from heat exchanger exit to reactor entrance is $100/20.9 \approx 5$, then the compressor proper must account for a compression ratio of $5/1.655 \approx 3$.

Now, the compression ratio per stage is likely to be about 1.25 so the number of stages, n, is found from the relationship

$$1.25^n \geq 3 \qquad 167$$

where n is an integer, in this case 5, indicating five stages.

The gas density at the heat exchanger exit is taken as $(2680/2075)29.6 = 38.2$ kg/m^3 (see equations 147 and 149). Let the intake velocity into the compressor be about one-half of the acoustic velocity at the inlet temperature or, roughly, 130 m/sec. Once again, applying the continuity equation, the cross sectional flow area in the compressor is found:

$$A = \frac{qM}{V\rho} = \frac{13.6x311}{130x38.2} = 0.85\ m^2 \qquad 168$$

The cross sectional flow area in an axial flow compressor is an annulus formed between the compressor casing and the hub to which the blades are attached. If the ratio of the blade tip to hub radius is selected as two, then this indicates the flow area will be an annulus formed between two circles, the larger one having a diameter of twice that of the smaller.

Geometric considerations will show, that for such a case, the radius of the larger circle, R_o, corresponding to a given area of the annulus is:

$$R_o = 2(A/3\pi)^{\frac{1}{2}} \qquad 169$$

For the problem under consideration the compressor radius would be:

$$R_o = 2(0.85/3\pi)^{\frac{1}{2}} = 0.6\ m \qquad 169a$$

This corresponds to a diameter of 1.2 m.

Now, each stage consists of a set of fixed blades and a set of rotating blades. Since the blade tip to hub radius ratio is two, then the hub radius is 0.6/2 = 0.3 m, and therefore the blade length is 0.6 - 0.3 = 0.3 m.

A reasonable axial length to radial length ratio for each blade is one to three. The axial length of each blade would thus be about 0.1 m or 10 cm. Allowing 3 cm for blade separation this indicates a stage length of 26 cm. The five stages would then come to a total of 5x26 = 130 cm or 1.3 m. Allow another 50 cm at each end for entrance and exit ducts and the dimensions of the compressor become, by a rough estimate, 1.2 m by 2.3 m. The compressor would have to be coupled to the driving turbine by a fluid coupling so as to ensure against gas losses.

About the only material the compressor could be constructed of would be tungsten or a tungsten alloy, unless it were cooled. Cooling the compressor (especially the rotor) would complicate matters considerably. Because of the centrifugal stresses involved, a blade tip to hub radius ratio of two might be unattainable at these high temperatures because of the lowered stresses permissible. In this case the compressor would have to be longer and narrower. If even this proved unfeasible, magnetic compression of the gas might have to be resorted to. However, this matter will not be pursued further here.

In order to reduce the fuel inventory in the compressor it might be tapered towards the exit end.

If the exit velocity of the gas is also taken as 130 m/sec the gas will still be $(130/705)^2 680 = 23$ degrees below the final temperature of 2320°K or at 2297°K. The corresponding pressure is $100(2297/2320)^{1.08/0.08}$ $= 88$ atm or $8.9\text{x}10^6$ newtons/m^2. The density at this point will then be:

$$\rho = \frac{8.9\text{x}10^6\text{x}3.11\text{x}10^2}{8.31\text{x}10^3\text{x}2.297\text{x}10^3} = 145 \text{ kg/m}^3 \qquad 170$$

Since the flow rate qM is 13.6x311 = 4230 kg/sec the cross sectional area of the duct will be:

$$A = \frac{4230}{130\text{x}145} = 0.224 \text{ m}^2 \qquad 171$$

corresponding to a duct diameter of 55.3 cm.

At the base of the reactor the gas will have to flow into some sort of a distributor, the function of which is to even the gas flow through the ducts in the reactor.

The duct would have to be constructed of graphite, perhaps with pyrographite lining, or tungsten. Close to the inlet end of the reactor there would be facilities for drawing off and replenishing a certain fraction of the gas for processing.

To save on fuel inventory, and if it should be necessary for maintaining subcritical conditions in the return line, it might prove necessary to return the gas at a low pressure to the vicinity of the reactor proper and place the compressor at the reactor inlet.

The length of the return line will of course be dependent on that of the generator. For a preliminary estimate (neglecting bend losses, etc.) assume that the maximum length of the return line is roughly equal to the sum of the lengths of the reactor, nozzle, generator (maximum), heat exchanger and compressor, or 1.045 + 2.014 + 38.4 + 1.37 + 2.3 = 45.13 m. At the Reynolds number corresponding to the gas flow conditions the friction factor will be about 10^{-3}. Hence, from equation 162, the approximate friction losses are:

$$w = 2\mathrm{x}10^{-3}\,\frac{1.3^2\mathrm{x}10^4\mathrm{x}45.13}{0.553} = 2.77\mathrm{x}10^3 \text{ joules/kg} \qquad 172$$

corresponding to a total friction loss rate of $13.6\mathrm{x}311\mathrm{x}2.77\mathrm{x}10^3 = 1.17\mathrm{x}10^7$ watts.

The 11.7 megawatts needed to overcome friction losses could be supplied by increasing the compressor rating, correspondingly, to 255.8 MW. The gas would then leave the compressor at a velocity of about 140 m/sec, and be slowed down to approximately 120 m/sec at the exit end, giving an average velocity of ≈130 m/sec in the return line.

On p. 101 the mean free path for fast neutrons, corresponding to a molal density of 0.0157 moles/m^3, was found to be of the order of 100 m. In the return line the molal density is 145/311 = 0.466 moles/m^3, indicating linear fast critical dimensions of the order of (0.0157/0.466)100 = 3.37 m.

In spite of the length of the duct, its diameter is an order of magnitude less than this, so the geometry will probably be subcritical. For a more accurate determination its fast buckling would have to be computed, and, should it prove supercritical, the pressure in the line would have to be reduced accordingly. If the pipe were to be constructed of graphite, it is not impossible that it might form a thermal "cavity" reactor. This will not be investigated here but if that were to prove to be the case, the graphite would have to be poisoned by a neutron absorbing material in order to keep it thermally subcritical.

The fuel inventory in the gas duct may be uncomfortably large. Making the most conservative assumption that the generator length will be a full 38.4 m and, consequently, the gas return line 45.13 m, the inventory would be $AL\rho$ = 0.224x45.13x145 = 1470 kg, which is indeed a sizeable quantity of a highly enriched gas.

As for the inventories of the heat exchanger and compressor, they would be 1.09x1.37x29.6 = 44.2 kg and (approximately) 2.3x0.85x65 = 127 kg, respectively. The value of 65 kg/m^3 was used as the estimated mean density in the compressor.

It is proposed to shield and cool the heat exchanger, compressor and gas return line in a similar fashion as the generator and core (see pp. 68 and 93).

The whole system is illustrated in Fig. 15. The total heat transfer surface, based on the outside area of the thermal insulation was computed from the dimensions of the various components (the detailed calculations will not be given here) and found to be approximately 456 m^2. If it is assumed that the heat loss rate per unit area is similar in the other parts of the system, as was calculated (equation 83) for the reactor, namely 8.23 KW/ft^2 or 85 KW/m^2, then the approximate heat loss rate from the whole system will be 456x85 = 38,700 KW. This corresponds to 3.8 percent of the heat released in the reactor and may be compensated for by increasing the gas flow rate accordingly.

IV. APPROXIMATE COST ESTIMATE

A. General Considerations

This chapter of the thesis must of necessity be very sketchy because of the inadequacy of the data available for the more unorthodox parts of the system. The estimate is carried out on a very general basis, and the procedure used the same as is commonly followed in elementary courses on cost analysis. This procedure gives adequately accurate preliminary cost estimates for conventional systems, and it is assumed that reasonably accurate values may be found for the system under consideration.

It is proposed to consider the whole generating system, illustrated in Fig. 15, as consisting of two parts. The reactor, nozzle, generator, return line and the thermal insulation and shielding external to the primary heat exchanger and compressor will constitute the first part.

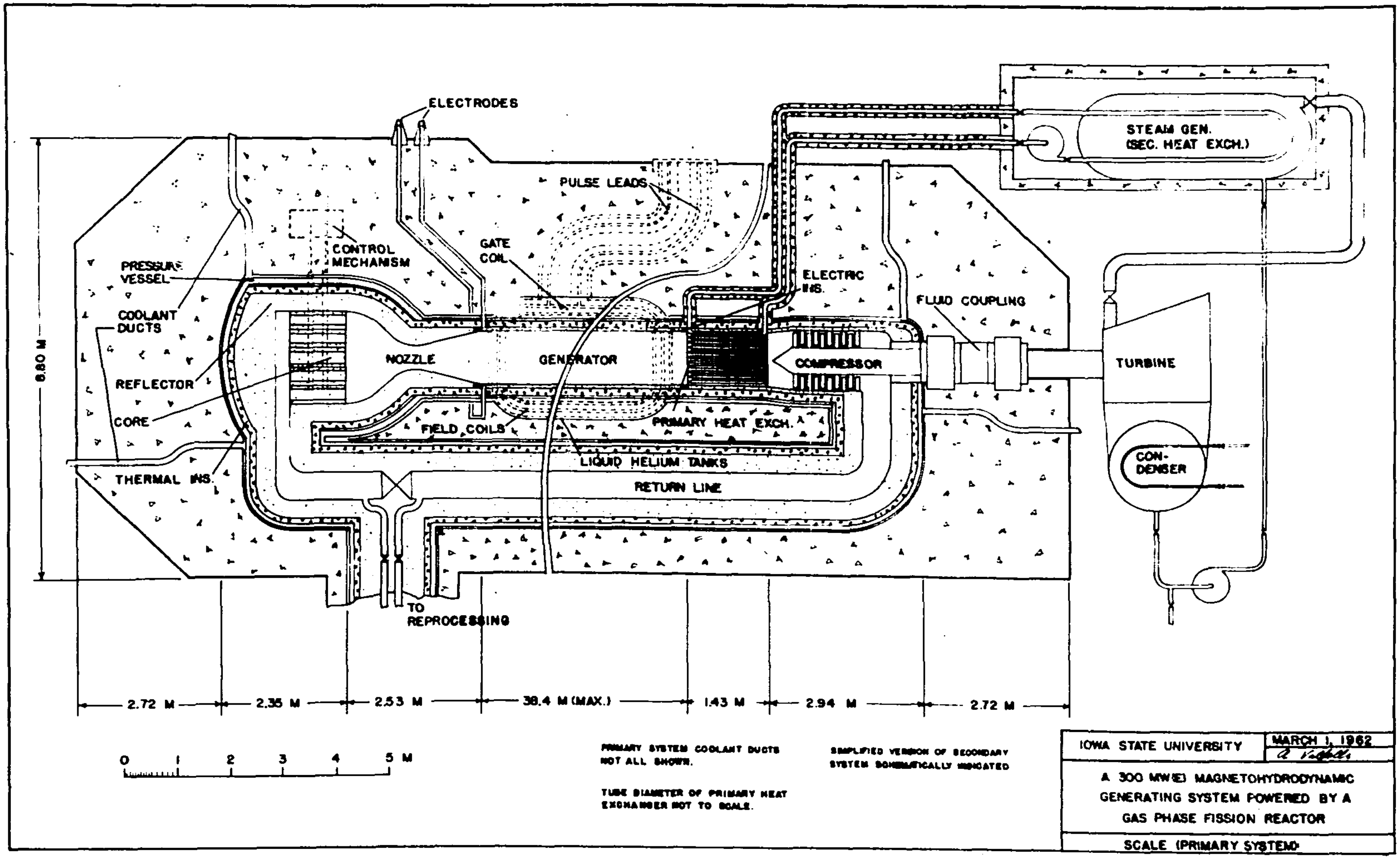
The second part will consist of the primary heat exchanger proper, the compressor and the rest of the secondary system.

This classification divides the system into essentially nonconventional and conventional parts.

B. Capital Costs

The capital cost estimate presented in this section will be based on the maximum length of the generator, i.e.,

Fig. 15. A proposed 300 MWe fissile fueled system for power generation by magnetohydrodynamic means



PRIMARY SYSTEM COOLANT DUCTS NOT ALL SHOWN.

TUBE DIAMETER OF PRIMARY HEAT EXCHANGER NOT TO SCALE.

SIMPLIFIED VERSION OF SECONDARY SYSTEM SCHEMATICALLY INDICATED

IOWA STATE UNIVERSITY

MARCH 1, 1962

A 300 MW(E) MAGNETOHYDRODYNAMIC GENERATING SYSTEM POWERED BY A GAS PHASE FISSION REACTOR

SCALE (PRIMARY SYSTEM)

that corresponding to the lowest theoretical gas conductivity.

As mentioned previously, the overall generating system will be considered to consist of "conventional" and "nonconventional" parts.

The quantities of the various materials of construction needed for the "nonconventional" system were determined from the design dimensions computed in the previous sections of the thesis. The detailed calculations, which consist of simple (but lengthy) solid geometrical mensuration computations, will not be given here, but the results are tabulated in Table 1. The calculations may be checked by reference to the dimensions indicated in Fig. 15.

As for the "conventional" part, its cost is assumed to be of the same order as that of the corresponding items in a conventional power station of the same power output. The power output of the "conventional" system is that required to recompress and recirculate the gas, this being less than the output of the magnetohydrodynamic generator of the "nonconventional" system.

It should be stressed that the assumption has been made that the turbine-compressor pair will cost about the same as a turbine-generator unit of the same power output. Also, that the unusual construction material of the primary heat exchanger (tungsten or an alloy, say a tungsten-tantalum

Table 1. Summary of costs for "nonconventional" system

System Part	Material							
	C m tons	SiC m tons	Steel m tons	Al m tons	W m tons	Conc. m^3	Nb-Sn m tons	He m^3
Core	11.27	9.78	24.40			275		
Nozzle	2.54	5.48	7.90			126		
Generator	8.20	43.40		15.8	0.28	2228	79.2	53.7
Heat Exch. Shielding	0.60	3.28		1.5		86		
Compressor Shielding	0.97	5.52	7.44			326		
Return Line	78.00	93.50	91.00		8.04			
Total	100.58	160.96	130.74	17.3	8.32	3041	79.2	53.7
Unit Cost (Fabricated material) K$	2.20	1.4	1.39	1.32	33.00	.244	55.0	17.0
Itemized Cost K$	233	226	182	24.8	274	744	4370	914
GRAND TOTAL			6968 K$		(K$ = Kilodollars)			

[a] Ellis, Tom, Dept. of Metallurgy, Iowa State University of Science and Technology, Ames, Iowa. Data concerning probable cost of superconductive alloys. Private communication. 1961.

alloy) will not be so costly as to cause a large error in lumping the heat exchanger together with the "conventional" system and assuming overall conventional costs of that system.

Table 1 lists the quantities and costs (see reference 34) corresponding to the maximum generator length. The grand total for the fabricated costs of the major components of the "nonconventional" system comes to \$6,968,000. This figure must be multiplied by some factor, f_f (for fudge factor), in order to obtain the final installed cost. For many industrial engineering installations, such as chemical plants and many types of nuclear reactors, this factor is in the neighborhood of 1.6. The final installed cost of the "nonconventional" system will thus be assumed to be 6,968,000x1.6 = \$11,150,000.

As for the "conventional" parts of the system, if a sodium primary heat transfer loop is decided upon, the costs will be assumed the same as those in a conventional reactor system employing sodium heat transfer loops (such as in the sodium graphite reactor system). Sargent and Lundy (35) have made extensive cost estimates for various types of reactor systems, including the sodium-graphite system. The capital costs are given for 75 MWe, 200 MWe and 300 MWe installations. Interpolation of their data gives the total capital cost of a 255 MWe plant of this type as \$80,000,000, with the reactor equipment proper costing $9,000,000. The non-nuclear part of the facilities is thus valued at some 80 - 9 = 71 M$ (M$ = megadollars).

Since, in the design under consideration, the "conventional" part would correspond to the non-nuclear part of an installation, such as mentioned above, its cost will also be taken as 71 M$. To this must be added the estimated capital cost of the "nonconventional" part of 11.15 M$, giving a total of some 82.15 M$.

It should be noted that the Sargent and Lundy figures correspond to two sodium loops employed in the heat transfer in a series fashion rather than the single loop indicated in Fig. 15. As two loops would probably be required because of radiological considerations the cost figures were employed directly.

C. Operating Costs

Classification of costs

The operating costs will be divided into two categories, namely, fixed charges and fuel costs. Under the former heading will be lumped maintenance (or depreciation), direct operating costs and insurance costs; in other words, all the costs of running the installation apart from those due to the fuel.

The fuel charges are further subdivided into the use charge, the burnup charge and the cost of fuel reprocessing.

Fixed charges

For most nuclear power plants the fixed charges amount to about 17 percent of the capital costs and this figure will be assumed to hold here. The annual basic fixed charges will then be 0.17x82.15 = 14 M\$. Apart from this, however, one must consider the additional charges incurred for this type of system, due to the losses of liquid helium employed for cooling the coils.

For the quantity of helium involved, it is probable that the losses may be kept down to less than one percent per day. With the present price of liquid helium at \$17,000/m^3 this would amount to some $0.01 \times 17{,}000 \times 53.7 \times 365 \simeq 3.33$ M\$. The total annual fixed charges would thus amount to 14 + 3.33 = 17.33 M\$.

Reprocessing costs

It should be noted that the fuel need not be fabricated into fuel elements. Therefore, the reprocessing costs will be relatively small. Transportation costs, which will be included under this heading, come to about \$10/kg U. Chemical processing for fission product removal costs \$20.40/kg U and reconversion to UF_4 approximately \$5.60/kg U. (Actually this last figure is the cost of converting the metal to the hexafluoride; conversion to the tetrafluoride would be slightly less than this.) The reprocessing costs thus come to a total of \$36/kg U.

Since 1 MWd is equivalent to 1.018 g U^{235} fissioned, the reactor, with its thermal power level of 1000 MW, will have a net consumption of 1.018 kg U^{235} per day. If the makeup gas is 90 percent enriched and, as the gas bled off for reprocessing is of the same enrichment as the reactor inventory gas, namely, 85 percent, then the consumption of makeup gas per day may be computed.

Ignoring the small mass defect and making a material balance on the basis of the U^{238} isotope being the invariant (corresponding to ashes in an ordinary combustion furnace), then let F designate the feed rate and W the removal rate of fuel, and, as before, the subscripts 25 and 28 the light and heavy isotope of uranium, respectively. Then, for the U^{238}:

$$W_{28} = F_{28} = F_{25}/9 = 0.111F_{25} \qquad 173$$

Since the enrichment of the waste is 85 percent, it follows that:

$$W_{25} = (85/15)W_{28} = 5.67W_{28}$$

$$= 0.111 \times 5.67F_{25} = 0.63F_{25} \qquad 174$$

A partial material balance on the U^{235} yields:

$$F_{25} = 1.018 + W_{25}$$

$$= 1.018 + 0.63F_{25} \qquad 175$$

Therefore,

$$F_{25} = \frac{1.018}{0.37} = 2.75 \text{ kg/day} \qquad 176$$

This corresponds to a total feed rate of 2.75/0.9 = 3.05 kg/day for the 90 percent enriched feed gas. Hence the reprocessing costs come to 3.05x36x365 = $40,150 per year.

Use charge

When equilibrium conditions have been reached in the reactor, there will be an appreciable quantity of fission products present, due to the relatively high burnup. Thus, the actual fuel inventory will be considerably less than that corresponding to the reference design. However, use charges must be paid on the original fresh fuel inventory, which is the same as that assumed in the reference design. Therefore, the inventories computed in Chapter III in this thesis are applicable to the use charge calculations.

The computed fuel inventories for the various parts of the system were as follows:

Core header	76.6 kg U	
Core	109.5 " "	
Nozzle	82.0 " "	(approx.)
Generator	353.0 " "	
Heat exch.	44.2 " "	
Compressor	127.0 " "	
Return line	1470.0 " "	
Total	2262.3 kg U	

The present cost of 85 percent enriched fuel is \$11,570/kg U (36), giving a total inventory value of 2262x11,570 = \$25,800,000 or 25.8 M\$. The present use charge charged by the AEC is 4.75 percent per annum (36), whereby the use charge on the fuel inventory becomes 25.8x0.0475 = 1.228 M\$.

In addition to this there is a quantity of fuel tied up in the cooling, transportation and reprocessing facilities. Allowing for a minimum cooling time of 60 days and transportation and reprocessing time of 90 days, or a total of 150 days, it is seen that the amount of fuel thus tied up is 150x3.05 = 452 kg U (3.05 kg is the daily feed rate of makeup gas). As the makeup gas is 90 percent enriched and thus valued at \$12,285/kg U the annual use charge on this quantity will be 452x12,285x0.0475 = \$264,000 or 0.264 M\$.

The total annual use charge is thus seen to be 1.228 + 0.264 = 1.492 M\$.

Burnup charge

The reactor uses 3.05 kg of 90 percent enriched uranium per day, or 3.05x365 = 1,110 kg annually. The value of this fuel is 1,110x12,285 ≈ 13.6 M$.

From equation 174, it is seen that the rejection rate of U^{235} is 0.63 times its feed rate of 2.75 kg/day, or 1.73 kg/day. Since the enrichment of the waste is 85 percent, this corresponds to 1.73/0.85 = 2.04 kg of uranium.

At $11,570/kg U this amounts to 2.04x365x11,570 ≈ 8.61 M$. The net burnup charge will therefore be 13.6 - 8.61 = 4.99 M$.

Power costs

If the installation is operated at a load factor of, say, 85 percent, the fixed charges and the use charge will remain unaffected, whereas the burnup charge and reprocessing cost will be reduced accordingly. The burnup charge would become 0.85x4.99 = 4.25 M$ and the reprocessing cost 0.85x0.04 = 0.0342 M$.

The total operating costs for a load factor of 85 percent may then be tabulated:

Fixed charges	17.330 M$
Repr. costs	0.034 "
Use charge	1.490 "
Burnup charge	4.250 "
Total	23.104 M$

For the load factor in question, the hours of operation per annum will be 0.85x24x365 = 7450. At the design power level of 3.04×10^8 watts, this corresponds to an annual energy output of $3.04\times10^8 \times 7.45\times10^3 = 2.26\times10^{12}$ watt hours or 2.26×10^9 kilowatt hours. Under the conditions specified in this section (i.e., maximum generator length, 85 percent load factor) the unit electrical cost would be $2.3104\times10^7/2.26\times10^9 = 0.0102$ \$/kw hr, or 10.2 mills per kilowatt hour.

V. DISCUSSION

Before proceeding further it might be useful to summarize some of the more pertinent parameters of the whole system:

Reactor:

Power	1000 MWt
Fuel	85% enriched UF_4 gas
Fuel cycle	Continuous circulation
Moderator	Graphite
Reflector	Graphite
Max. core temp.	3000° K
Flux	7.53×10^{14} n/cm^2 sec (initial)
Excess reactivity	0.24 (initial)
Burnup	33.3%
Control	HfC in graphite rods
Core geometry	Cylindrical
Core dimensions	Dia.: 1.88 m, height: 1.045 m

Generator:

Type	MHD
Power	304 MWe
Dimensions	1.14x1.14x38.4 m

General:

Shielding	Barytes concrete
Overall fuel inventory	2262 kg U
Overall efficiency	30.4%
Secondary system power	255 MW
Heat transfer agent	Liquid Na
Est. power generating cost	10.2 mills/kw hr

It is important to note that the generating cost of 10.2 mills per kilowatt hour is at best only a tentative estimate, because of the generality of much of the data it had to be based on. It is probable, though, that the generating cost would fall somewhere near this figure with a considerable leeway for probable error. About all that can be stated with a reasonable certainty is that the generating costs for the rather unorthodox type of system that this thesis describes are likely to be of a similar magnitude as those of conventional reactor systems.

Another point which merits further attention is the flux level. For the initial fresh fuel inventory with 85 percent enriched fuel and in the absence of fission products it would have to be $7.53\text{x}10^{14}$ n/cm^2 sec for a power level of 1000 MW, as previously computed. The rejection rate of spent fuel is 3.05 kg U/day (the same as the feed rate, except for the minute mass defect) and of this, 1.018 kg have been converted to fission products, corresponding to a burnup of 33.3 percent. As the composition of the rejects is the same as that of the inventory when equilibrium conditions have been attained, the fuel will only contribute 66.6 percent to the material within the reactor, the rest being fission products. Thus for an unchanged power output, the flux at equilibrium must be $7.53\text{x}10^{14}/0.666 = 1.13\text{x}10^{15}$ n/cm^2 sec. This represents a very high flux, but for the reasons stated previously (in the section on flux) it is not considered excessive.

As a matter of fact, if the flux could safely be increased considerably above this value, the fuel enrichment might be reduced accordingly. Were it possible to raise the flux level by a factor of ten, the enrichment might be lowered by the same factor for an unchanged power output (Allowances might have to be made for the decrease in the infinite multiplication factor though, by increasing the core size). This would cut the use charge by roughly the same factor, and, as the use charge contributes about 6.5 percent to the total generating costs, this would result in some reduction of power costs. The reduction would not only be direct, but would also stem from the plutonium credit to be gained by use of relatively low enrichment fuel. The burnup charge would not be materially affected. It is seen though, that even a strong flux increase affects the generating costs but slightly.

If, on the other hand, a flux of 1.13×10^{15} n/cm^2 sec should prove excessive, the core will have to be enlarged in order to accommodate a greater fuel inventory to allow a lower flux level. Since the core inventory is only a small part of the total inventory, the effect of this on generating costs would not be very great, either.

In other words, the power costs are not influenced greatly by the flux level.

The high burnup will result in a considerable poisoning effect. The extent of the poisoning was not investigated quantitively in the section on design, but it was assumed that the excess reactivity would be adequate to compensate for it. Still, it is a point which merits consideration.

The chief offenders are xenon-135 and samarium-149. If the poisoning effect of these two fission products is amply counteracted by excess reactivity, the presence of other fission products can generally be ignored.

During operation the xenon poisoning will come close to its limiting value of 0.05, because of the high flux. The limiting equilibrium value for samarium-149 is 0.012, giving a total negative reactivity equivalent of 0.062.

After shutdown the xenon-135 concentration will, at the high flux level of operation, increase very considerably. The poisoning for some time after shutdown (the xenon-135 decays) will probably be far greater than the excess reactivity of the reactor, thus necessitating a waiting period prior to renewed operation.

As for the samarium, its poisoning value after shutdown does not go through a maximum as is the case with the xenon, but tends to a value given by the equation:

$$\begin{aligned} P_\infty &= 1.5\times10^{-16}\phi + 0.012 \\ &= 1.5\times10^{-16}\times1.13\times10^{15} + 0.012 \\ &= 0.182 \end{aligned} \tag{177}$$

The excess reactivity of the reactor, which was computed to be 0.24, is enough to overcome the combined effect of samarium-149 and xenon-135 at equilibrium, and the effect of these same poisons at startup if enough time has been allowed to elapse since shutdown for the xenon-135 poisoning to have reached a value of about 0.05 (This leaves some allowance for other fission products.). The time required for allowing sufficient decay of the xenon-135 will not be computed here, but tentatively it can be estimated at a number of days (after a complete shutdown).

As the preceding cost estimate was based on the maximum generator length, it will be interesting to investigate how power costs would be affected if the generator could be made shorter.

The factors dependent on the generator length are:

a) Capital cost of generator
b) Cost of helium inventory
c) Cost of return line
d) Annual cost of helium losses
e) Use charge on fuel inventory in generator and return line

The cost of the generator is not proportional to its length alone, due to what might be termed "end effects", that is to say, that part of the field coils, (and the associated helium and tanks) which completes the electrical circuit at each end, i.e., the short sides of each rectangular turn.

On the same basis as in Table 1, the length dependent costs (installed) of the generator and helium were computed to be 9.05 M$ and those of the return line 0.90 M$, giving a total of 9.95 M$. Since this corresponds to a generator length of 38.4 m, the savings incurred by a shorter generator, of length L, would be ((38.4 - L)/38.4)9.95 M$. If it should turn out, that, for example, the electron scatter cross section of the uranium tetrafluoride molecule should only be about one-fourth of the maximum theoretical value (as is the case for some other tetrahalides), then this would be reflected in a generator length of only one-fourth as long as otherwise. The corresponding savings in capital costs would be (3/4)9.95 = 7.45 M$. At 17 percent fixed charges this carries to annual savings of 1.27 M$.

As for the helium, 94.7 percent of it is associated with that part of the field coils which is dependent on the generator length (the long sides of the rectangular turns) or 0.947x53.7 = 50.7 m^3, thus leaving 3 m^3 which are needed, regardless of the generator length. If the generator length can be reduced by a factor of four the total helium inventory would be reduced to 3 + (1/4)50.7 = 15.7 m^3. This represents (15.7/53.7)100 = 29.2 percent of the helium inventory that corresponds to the maximum generator length. Therefore, assuming an unchanged loss rate, the savings in helium losses per year would be (1.00 - 0.292)3.33 = 2.36 M$ (3.33 M$/yr was the cost of annual helium losses for 53.7 m^3 of helium.).

Finally, the use charge savings due to a reduced fuel inventory must be considered. The generator inventory for a generator length of 38.4 m was found to be 353 kg U. The total length of the return line is 47.96 m, of which 38.4 m is due to the generator. Therefore, the inventory in that portion of the return line which passes the generator is (38.4/47.96)1470 = 1175 kg U. The total fuel inventory dependent upon the generator length is thus found to be 353 + 1175 = 1528 kg U. As the total inventory of 2262 kg U represented a use charge of 1.49 M$, the savings incurred by reducing the generator dependent inventory by three-fourths, or (3/4)1528 = 1148 kg U, would be (1148/2262)1.49 = 0.755 M$.

Hence, the total annual savings obtained by shortening the generator by three quarters would be those due to the reduction in fixed charges, the decrease of the helium losses and the diminished use charge, or 1.27 + 2.36 + 0.755 = 4.385 M$/yr. At a load factor of 85 percent this is equivalent to a savings of 1.9 mills/kw hr, thus reducing the generating costs to 8.3 mills/kw hr. It should be kept in mind though that this is based on the (speculative) assumption that the electron scatter cross section for uranium tetrafluoride has a value of about one-fourth of its theoretical maximum. An experimental measurement of its magnitude would be required for any accurate prediction of the generator length and its effect on power costs.

Another factor which affects the generator length strongly, is the strength of the magnetic field. Quite recently there have been reports of a gallium-vanadium alloy that retains its superconductivity at field strengths of up to 500 kilogauss (37). This is a fivefold improvement over the niobium-tin alloy on which the present design is based, and, if the former type of alloy were to be employed, the generating costs might be cut considerably. However, that point will not be pursued further here.

The secondary system which drives the compressor has a power output of 255 MW. This is derived from the 696 MW rejected by the primary system and it follows that the efficiency of the secondary system must be 255/696 = 0.366. Considering the very high temperature at which heat is supplied to the secondary system (i.e., rejected by the primary system) this degree of efficiency is nothing out of the ordinary. It is quite possible that this level of efficiency could be maintained even if the heat sink of the secondary system were to be at a relatively high temperature, say, between 100° C and 200° C. If this could be done the heat rejected by the secondary system could be sold as process heat to a chemical plant. For example, a heavy water plant uses heat at the rate of $6x10^6$ Btu/lb D_2O. Since the reactor installation would reject heat at the rate of 700 - 255 = 445 MW, it would reject a total of $445x7.45x10^3$

$= 3.32\text{x}10^6$ megawatt hours annually (at a load factor of 85 percent). This is equivalent to $3.32\text{x}10^6\text{x}3.412\text{x}10^6 = 11.3\text{x}10^{12}$ Btu/yr) which is sufficient to supply a heavy water plant with an annual capacity of $11.3\text{x}10^{12}/6\text{x}10^6 = 1.89\text{x}10^6$ lb/yr or 945 tons per year. The Savannah River plant, by comparison, has a capacity of 500 tons per year. If this power could be sold at, say, $0.25/10^6 Btu, the additional revenue from the rejected heat would be $0.25\text{x}11.3\text{x}10^6 \approx 2.82$ M$/yr. This corresponds to about 1.2 mills for each kilowatt hour of electrical energy the installation produces, and, if need be, the power costs could be lowered accordingly.

Another approach to lowering capital costs would entail lowering the cost of the "conventional" part of the system. One possibility might be to utilize an organic coolant instead of a liquid metal such as sodium. The maximum temperature of the organic coolant would have to be much lower, however, than that allowable with liquid sodium. One consequence of this would probably be that in order to obtain the required efficiency of the secondary system heat could not be rejected at such a high temperature as to have a value as process heat.

The Handbook of Chemistry and Physics (38) lists picene as the organic compound with the highest boiling point, namely, 520° C. Picene is a polynuclear hydrocarbon quite similar in molecular structure (dibenzo(a,i)phenanthrene) to the more conventional organic coolants, such as the constituents of Santowax R. Consequently, picene might be expected to have similar properties and be useful as a coolant.

Assuming then, that an organic coolant could be employed in the secondary system and, furthermore, that the costs of such a secondary system would be approximately the same as that of the equivalent component of an organically cooled conventional nuclear power plant, the approximate effect on the capital costs can be estimated.

Interpolation of data from Sargent and Lundy (35) gives the cost of a 255 MWe organic cooled nuclear power installation as 56.5 M$. The cost of the reactor equipment in such a plant was similarly found to be 5.5 M$ giving the cost of the nonnuclear part of the installation as 56.5 - 5.5 = 51 M$. The corresponding cost for the liquid sodium cooled plant was 71 M$. Thus the capital costs of an organic cooled facility would be lower by some 20 M$. At 17 percent fixed charges per annum this comes to a yearly savings of 3.4 M$. This corresponds to a savings of about 1.4 mills per kilowatt hour.

In summary then, the probable power generating costs with a sodium cooled facility are estimated to be 10.2 mills/kw hr. It is conceivable that the generator might be shortened to such an extent as to lower the costs to 8.3 mills per kilowatt hour. If the rejected heat could be sold, this would entail a savings of 1.2 mills/kw hr, which, together with a short generator, might give costs as low as 7.1 mills/kw hr (This is probably overly optimistic, though.).

If the reactor were to be organically cooled, the power costs for a full length generator could be expected to be lower than those of a liquid sodium cooled facility by 1.4 mills/kw hr corresponding to a unit power cost of 8.8 mills/kw hr. The corresponding cost for the shorter generator length would be 8.3 - 1.4 = 6.9 mills/kw hr.

The last combination (even if the generator could not be shortened to one-fourth of its maximum theoretical length) seems to give the lowest power costs. It would be logical then, to use an organic coolant rather than a liquid metal.

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VII. ACKNOWLEDGEMENTS

First and foremost, the author would like to thank his major professor, Dr. Glenn Murphy, Head of the Department of Nuclear Engineering at Iowa State University. Were it not for Dr. Murphy's encouragement and advice offered in connection with the topic of this thesis, the author probably never would have undertaken the project.

Thanks are also due to Drs. L. H. Burkhart and T. Wheelock of the Department of Chemical Engineering, as well as Drs. P. H. Carr and C. A. Swenson of the Department of Physics, for assistance rendered.

Finally, the author would like to thank Miss Joanne Clarke, who, apart from typing the thesis, aided him extensively in the preparation of its actual physical format.

VIII. APPENDIX

A. Electron Scatter Cross Section Measurements

Although electron scatter cross section values for many tetrahalides have been published, no data for uranium tetrafluoride was found in the literature.

Therefore, for a more accurate generator design, the cross section of the uranium tetrafluoride molecule will have to be determined experimentally.

The method conventionally employed involves sending a beam of electrons (having the desired energy) through a space which can be filled with the vapour of the compound in question.

By collecting the beam on an electrode the beam current in the absence and the presence of the vapour can be measured and the attenuation due to the vapour may thus be determined.

Let j_0 signify the current density in the absence of the vapour, N its molecular density and σ the cross section. Then the current density in the presence of the vapour, j, is given by the relation:

$$j = j_0 e^{-N\sigma x} \qquad 178$$

If the electron beam is formed by an oxide coated cathode such as in a cathode ray tube (the electrons may be accelerated by a grid to the velocity corresponding to the desired electron energy), the electron velocities will have a maxwellian spread corresponding to the cathode temperature. For greater accuracy a beam of more nearly monoenergetic electrons may be formed by using Ramsauer's method of achieving this (See Ref. 19, p. 4, Ref. 39, pp. 39, 393).

As uranium tetrafluoride is solid at ordinary temperatures the vapour containing space would have to be heated in order to prevent condensation of the vapour. At 792°C the vapour pressure of uranium tetrafluoride is 10^{-5} atm, which should be adequate for attenuation measurements. However, if only the vapour space is heated, this may give rise to effects that would disturb the desired measurements, such as thermionic and thermoelectric effects. Because of these difficulties it might be necessary to resort to other methods of determining the conductivity of uranium tetrafluoride gas. Perhaps the most direct method would be a simple measurement of the conductivity of the vapour (possibly laced with cesium fluoride to provide electrons) in a conductivity cell operated at a high temperature.

B. Table of Nomenclature

The MKS system of units was generally employed throughout the thesis, although often used interchangeably with the cgs system. Occasionally, the engineering system of British units (the fps system) was resorted to where the computation data warranted it. The units used were specified in each case, so they will not be noted for each symbol listed below.

Because the thesis touches on many fields, and the nomenclature conventional to each field was generally retained, it will be found that the same quantity may be designated by different symbols in different parts of the thesis, and vice versa. This was found preferable to using the same symbols for the same quantity consistently, as the thus unfamiliar connection between symbol and quantity would probably be of hindrance to readers acquainted with the customary nomenclature in each field.

Vector quantities are designated by an arrow, otherwise magnitude alone is indicated, except where the vector property is implicit, as for example, following a ∇ sign:

A - area.

a - velocity of sound, molecular or orbital radius.

B - magnetic flux density, buckling.

C - heat capacity per mole.

c - heat capacity per unit weight.

D - diameter.

E - energy, electric potential.

E_o - most probable energy, electrode potential.

F - free energy per mole, force, feed rate.

f - thermal utilization, friction factor, frequency.

g - statistical weight factor, acceleration of gravity.

H - enthalpy per mole, magnetic intensity.

h - enthalpy per unit weight, Planck's constant.

$\hbar$ - Planck's constant/2π.

i - electric current.

j - electric current density.

K - chemical equilibrium constant.

k - multiplication factor, thermal conductivity, Boltzman constant.

L - diffusion length, electrode separation.

M - molecular weight.

m - electronic or molecular mass.

N - number of atoms.

$\bar{N}$ - atomic density.

N_{Av} - Avogadro's number.

n - molecular density, number of turns or stages.

P - vapour pressure, power, poisoning.

P_t - total power dissipation.

p - pressure.

Pr - Prandtl number.

Q - heat.

q - heat transfer per unit area, flow rate.

R - universal gas constant, radius.

Re - Reynold's number.

r - radius.

S - entropy per mole, stress, source strength.
T - temperature.
T_t - transition temperature.
t - time.
U - heat of ionization per mole.
V - velocity.
v - velocity.
W - waste rate.
W_n - net work.
w - net work per unit weight.
x - fraction of.

Greek symbols:

α - rate constant.
β - volume coefficient of expansion.
γ - ratio of specific heats.
Δ - differential.
δ - reflector savings.
η - conductivity.
θ - deflection angle.
κ - inverse diffusion length.
λ - relaxation length.
μ - magnetic permeability, viscosity.
ν - collision frequency.
ξ - mean logarithmic energy decrement per collision.
Π - total pressure.

ρ - density.

Σ - macroscopic cross section.

σ - microscopic cross section.

τ - Fermi age.

ϕ - neutron flux.

ψ - wave amplitude.

ω - gyromagnetic frequency.

Subscripts:

a - indicates area.

e - indicates electron.

eff - indicates effective.

f - indicates fission.

g - indicates geometric.

i - indicates ion.

M - indicates molecular.

m - indicates material.

p - indicates at a constant pressure, poison.

r - indicates reflector.

T - indicates at a temperature of.

t - indicates nozzle throat.

v - indicates at a constant volume.

x - indicates nozzle exit.

0 - indicates fuel, at the most probable energy.

1 - indicates moderator.

∞ - indicates infinite.

$+$ - indicates positive ion.

$-$ - indicates electron.